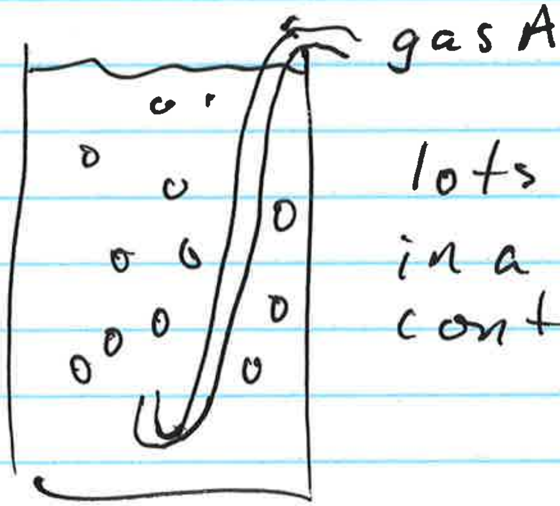
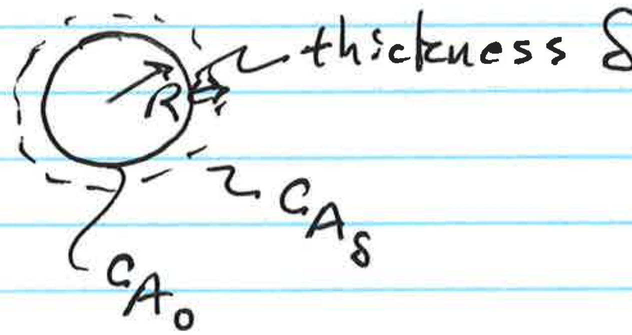
POD '19

Gas absorption in a sparger



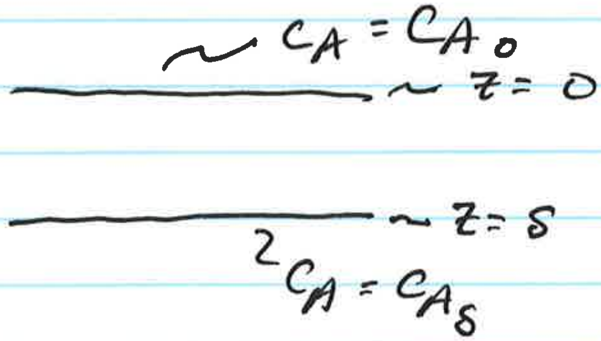
lots of bubbles in a well-stirred container



Gas A dissolves at surface & diffuses & reacts away in liquid B.

Usually we look at the case $\delta/R \ll 1$ (thin diffusional layer).

In this case the solution to the conc. profile is the <u>same</u>: "flat-earth" as would get from cartesian coord:



As before, $\mathcal{D}_{AB} \dfrac{\partial^2 C_A}{\partial z^2} = k_1 C_A$

Let $z^* = \dfrac{z}{\delta}$, $C_A^* = C_A / C_{A_0}$

$\therefore \dfrac{\mathcal{D}_{AB}}{\delta^2} \dfrac{\partial^2 C_A^*}{\partial z^{*2}} = k_1 C_A^*$

so $b_1 \equiv \left(\dfrac{k_1 \delta^2}{\mathcal{D}_{AB}} \right)^{1/2} = \text{Hatta}$ *

and $\dfrac{\partial^2 C_A^*}{\partial z^{*2}} = b_1^2 C_A^*$

B.C.'s : $C_A^* \Big|_{z^*=0} = 1$, $C_A^* \Big|_{z^*=1} = \dfrac{C_{A_\delta}}{C_{A_0}}$

so $C_A^* = A \sinh b_1 z^* + B \cosh b_1 z^*$

$C_A^* \Big|_{z^*=0} = 1 \therefore B = 1$

Now $C_A^*\Big|_{z^*=1} = \frac{C_{A\delta}}{C_{A_0}} = A \sinh b_1 + \cosh b_1$

$\therefore A = \left(\frac{C_{A\delta}}{C_{A_0}} - \cosh b_1\right)\frac{1}{\sinh b_1}$

and $C_A^* = \frac{\sinh b_1 z^*}{\sinh b_1}\left(\frac{C_{A\delta}}{C_{A_0}} - \cosh b_1\right) + \cosh b_1 z^*$

The flux at $z^*=0$ is:

$$N_{A_z}\Big|_{z=0} = -\mathcal{D}_{AB}\frac{\partial C_A}{\partial z}\Big|_{z=0} = -\frac{\mathcal{D}_{AB} C_{A_0}}{\delta}\frac{\partial C_A^*}{\partial z^*}\Big|_{z^*=0}$$

$$= -\frac{\mathcal{D}_{AB} C_{A_0}}{\delta}\frac{b_1}{\sinh b_1}\left(\frac{C_{A\delta}}{C_{A_0}} - \cosh b_1\right)$$

To close we need $C_{A\delta}$. We've assumed SS, so the flux at $z=\delta$ must be consumed in the rest of the volume!

Suppose we have an area A of bubbles in a volume V of our tank.

If the total volume of the bubbles is small (and $A\delta << V$ is even smaller)

then
$$-A\,\mathcal{D}_{AB}\left.\frac{\partial c_A}{\partial z}\right|_{z=\delta} = V k_1 c_{A_\delta}$$

or
$$\frac{c_{A_\delta}}{c_{A_0}} = -\frac{A}{V}\frac{\mathcal{D}_{AB}}{k_1\delta}\left.\frac{\partial c_A^*}{\partial z^*}\right|_{z^*=1}$$

$$= -\frac{A\delta}{V}\frac{1}{b_1^2}\left.\frac{\partial c_A^*}{\partial z^*}\right|_{z^*=1}$$

Now
$$\left.\frac{\partial c_A^*}{\partial z^*}\right|_{z^*=1} = \frac{b_1\cosh b_1}{\sinh b_1}\left(\frac{c_{A_\delta}}{c_{A_0}} - \cosh b_1\right) + b_1\sinh b_1$$

This yields
$$\frac{c_{A_\delta}}{c_{A_0}} = \frac{1}{\cosh b_1 + b_1\left(\frac{V}{A\delta}\right)\sinh b_1}$$