POD '18

Effectiveness factor for flat particles

$$N_{A_y} = -c\,\mathcal{D}_A \frac{\partial x_A}{\partial y}$$

$$\frac{\partial N_{A_y}}{\partial y} = -R_A = -k_1 a C_A$$

$$\therefore \quad c\,\mathcal{D}_A \frac{\partial^2 C_A}{\partial y^2} = k_1 a C_A$$

$$C_A\Big|_{y=b} = C_{A_0}$$

$$\frac{\partial C_A}{\partial y}\Big|_{y=0} = 0$$

Scale: $C_A^* = C_A / C_{A_0}$ $\quad y^* = y/b$

$$\therefore \frac{\partial^2 C_A^*}{\partial y^{*2}} = \left(\frac{k_1 a b^2}{\mathcal{D}_A}\right) C_A^* \qquad = \Lambda^2 C_A^*$$

$$C_A^* = A \sinh \Lambda y^* + B \cosh \Lambda y^*$$

$$\left.\frac{\partial C_A^*}{\partial y^*}\right|_{y^*=0} = 0 \quad \therefore \quad A = 0$$

$$C_A^*\Big|_{y^*=1} = 1 \quad \therefore \quad B = \frac{1}{\cosh \Lambda}$$

$$\text{so} \quad C_A^* = \frac{\cosh \Lambda y^*}{\cosh \Lambda}$$

$$N_A\Big|_{y=b} = -\mathcal{D}_A \left.\frac{\partial C_A}{\partial y}\right|_{y=b} = -\frac{\mathcal{D}_A C_{A_0}}{b} \left.\frac{\partial C_A^*}{\partial y^*}\right|_{y^*=1}$$

$$= -\frac{\mathcal{D}_A C_{A_0}}{b} \Lambda \tanh \Lambda$$

$$\text{As } \Lambda \to 0 \quad N_A\Big|_{y=b} = -\frac{\mathcal{D}_A C_{A_0}}{b}\left(\frac{k_1 a b^2}{\mathcal{D}_A}\right)$$

$= -C_{A_0} k_1 a b$

So $\dfrac{N_A\big|_{y=b}}{N_A\big|_{y=b}\big|_{\phi \to 0}} = \dfrac{\tanh \phi\Lambda}{\phi\Lambda} \equiv \eta_A$

Now for a flat particle

$\dfrac{V_p}{S_p} = b$ (vol/area)

$\therefore \Lambda_{fp} = \left(\dfrac{k_1 a}{D_A}\right)^{1/2} \dfrac{V_p}{S_p}$

So $\eta_A = \dfrac{\tanh \Lambda}{\Lambda}$

vs. for a sphere $\Lambda = \dfrac{\phi}{3}$

$$\eta_A = \frac{3}{3\Lambda^2}\left(3\Lambda \coth 3\Lambda - 1\right)$$

we can plot these up for comparison

Plot of Effectiveness Factor

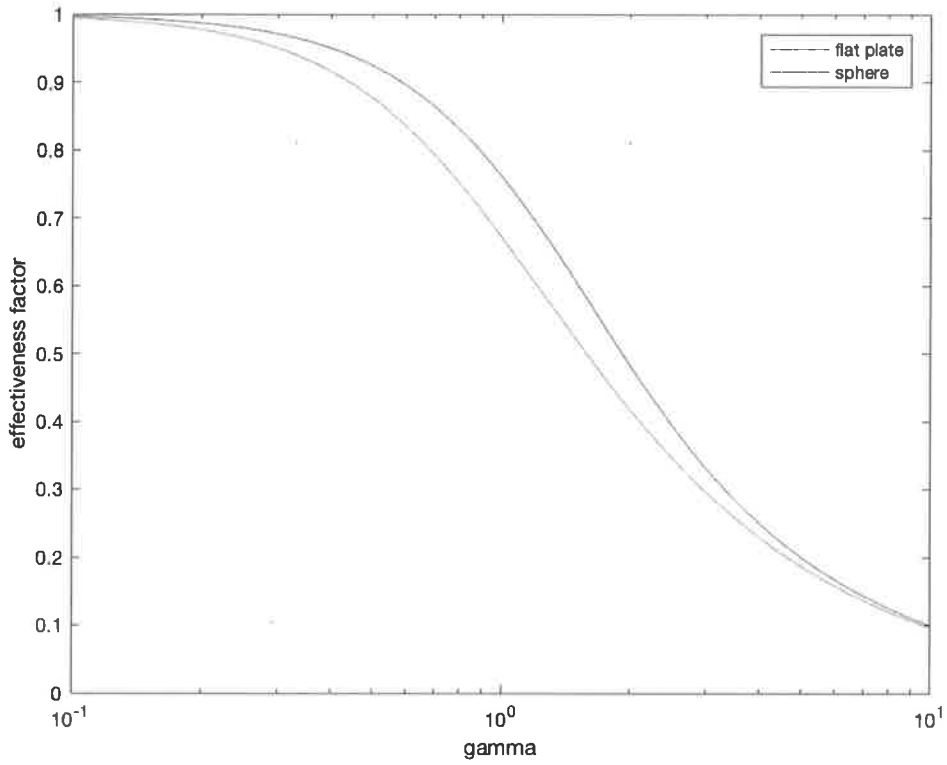
We compare the Thiele Effectiveness Factor for diffusional limitations of catalysis for the sphere and flat plate (two limiting shapes).

```
gam = [.1:.01:10];

etafp = @(x) tanh(x)./x;

etasph = @(x) (3*x.*coth(3*x)-1)./x.^2/3;

figure(1)
semilogx(gam,etafp(gam),gam,etasph(gam))
xlabel('gamma')
ylabel('effectiveness factor')
legend('flat plate','sphere')
grid on
```



Published with MATLAB® R2017a