Solution to 1^{st} order linear ODE's

If we have the DE

$$\frac{dy}{dx} + P(x)\,y = f(x)$$

Then $$y = e^{-\int P(x)\,dx}\left[\int f(x)\,e^{\int P(x')\,dx'}\,dx + K\right]$$

where K is determined from IC

Example:

$$\frac{dy}{dx} - \frac{ay}{x^2} = \frac{b}{x^2}$$

$$\therefore\quad P(x) = -\frac{a}{x^2}\,,\quad f(x) = \frac{b}{x^2}$$

$$\int P(x)\,dx = \int -\frac{a}{x^2}\,dx = \frac{a}{x}$$

$$\therefore\quad y = e^{-\frac{a}{x}}\left[\int \frac{b}{x^2}\,e^{\frac{a}{x}}\,dx + K\right]$$

$$= e^{-\frac{a}{x}}\left[-\frac{b}{a}\,e^{\frac{a}{x}} + K\right]$$

$$= -\frac{b}{a} + K\,e^{-\frac{a}{x}}$$

Dittus-Boelter Correlation (and hydraulic radius)

$$Nu = 0.023\, Re^{4/5}\, Pr^{n}$$

All parameters are evaluated at the <u>average</u> <u>bulk</u> <u>temperature</u>

$n = 0.3$ for <u>cooling</u>

$n = 0.4$ for <u>heating</u>

$$Re \equiv \frac{U D_H}{\nu} \qquad Nu \equiv \frac{h D_H}{k}$$

where D_H is the <u>hydraulic</u> <u>diam.</u>

$$D_H = \frac{4A}{P}$$

$A \leftarrow$ x-sectional area

$P \leftarrow$ wetted perimeter

For an annular pipe:

$$A = \frac{\pi}{4}\left(D_1^2 - D_0^2\right)$$

$$P = \pi\left(D_1 + D_0\right)$$

$$\therefore\ D_H = \frac{D_1^2 - D_0^2}{D_1 + D_0} = D_1 - D_0$$