$$\frac{\partial u^*}{\partial x^*} + \frac{\partial v^*}{\partial y^*} = 0$$

$$\frac{1}{Pr}\left(u^*\frac{\partial u^*}{\partial x^*} + v^*\frac{\partial u^*}{\partial y^*}\right) = \frac{\partial^2 u^*}{\partial y^{*2}} + T^*$$

$$u^*\frac{\partial T^*}{\partial x^*} + v^*\frac{\partial T^*}{\partial y^*} = \frac{\partial^2 T^*}{\partial y^{*2}}$$

$$T^*\Big|_{y^*=0} = 1 \qquad T^*\Big|_{y^*\to\infty} = 0$$

$$u^*\Big|_{y^*=0} = 0 \qquad u^*\Big|_{y^*\to\infty} = 0$$

$$v^*\Big|_{y^*=0} = 0$$

Stretching:

$$u^* = A\bar{u}, \quad v^* = B\bar{v}, \quad T^* = C\bar{T}, \quad x^* = D\bar{x}, \quad y^* = E\bar{y}$$

From BC: $C\bar{T}\Big|_{\bar{y}=0} = 1 \quad \therefore C = 1$

CE: $$\frac{A}{D}\frac{\partial \bar{u}}{\partial \bar{x}} + \frac{B}{E}\frac{\partial \bar{v}}{\partial \bar{y}} = 0$$

$$\therefore \frac{EA}{BD} = 1 \qquad B = A\frac{E}{D}$$

Energy: $$\frac{CA}{D}\left(\bar{u}\frac{\partial \bar{T}}{\partial \bar{x}} + \bar{v}\frac{\partial \bar{T}}{\partial \bar{y}}\right) = \frac{C}{E^2}\frac{\partial^2 \bar{T}}{\partial \bar{y}^2}$$

So: $$\frac{E^2 A}{D}\left(\bar{u}\frac{\partial \bar{T}}{\partial \bar{x}} + \bar{v}\frac{\partial \bar{T}}{\partial \bar{y}}\right) = \frac{\partial^2 \bar{T}}{\partial \bar{y}^2}$$

$$\therefore \frac{E^2 A}{D} = 1$$

Momentum:

$$\frac{A^2}{D}\frac{1}{Pr}\left(\bar{u}\frac{\partial \bar{u}}{\partial \bar{x}} + \bar{v}\frac{\partial \bar{u}}{\partial \bar{y}}\right) = \frac{A}{E^2}\frac{\partial^2 \bar{u}}{\partial \bar{y}^2} + C\bar{T}$$

$$\therefore \frac{AE^2}{D}\frac{1}{Pr}\left(\bar{u}\frac{\partial \bar{u}}{\partial \bar{x}} + \bar{v}\frac{\partial \bar{u}}{\partial \bar{y}}\right) = \frac{\partial^2 \bar{u}}{\partial \bar{y}^2} + \frac{E^2 C}{A}\bar{T}$$

$\therefore \frac{AE^2}{D} = 1$ (again) and $\frac{E^2 C}{A} = 1$

So: $B = A\frac{E}{D}$, $C = 1$

$$\frac{E^2 A}{D} = 1 \qquad \frac{E^2 C}{A} = 1$$

Put all complexity in D: stretching param. for x!

So: $\boxed{C = 1}$; $\frac{E^2}{A} = 1 \therefore A = E^2$

$$\therefore \frac{E^2 A}{D} = \frac{E^4}{D} = \boxed{\frac{E}{D^{1/4}} = 1}$$

$$E = D^{1/4} \quad \therefore \frac{E^2}{A} = \frac{D^{1/2}}{A} = \boxed{\frac{A}{D^{1/2}} = 1}$$

$$\frac{BD}{AE} = 1 = \frac{B\,D}{D^{1/2}\,D^{1/4}} = \boxed{BD^{1/4} = 1}$$

$$\therefore \frac{u^*}{x^{*\,1/2}} = f(\eta)\,; \quad v^* x^{*\,1/4} = g(\eta)$$

$$T^* = h(\eta) \qquad \eta = \frac{y^*}{x^{*\,1/4}}$$

Now for the DE's!

$$\frac{\partial u^*}{\partial x^*} = \frac{1}{2} x^{*\,-1/2} f - \frac{1}{2} x^{*\,1/2} \frac{\eta}{x^*} f'$$

$$= \frac{1}{2} x^{*\,-1/2} (f - \eta f')$$

$$\frac{\partial v^*}{\partial y^*} = x^{*\,-1/2} g'$$

$$\therefore \text{CE:} \quad \boxed{\begin{aligned} g' &= -\frac{1}{2}(f - \eta f') \\ g(0) &= 0 \end{aligned}}$$

Energy: $\dfrac{\partial T^*}{\partial y^*} = x^{*-1/4} h'$

$$\frac{\partial^2 T^*}{\partial y^{*2}} = x^{*-1/2} h''$$

$$\frac{\partial T^*}{\partial x^*} = -\frac{1}{4} \frac{\zeta}{x^*} h'$$

$$u^* = x^{*1/2} f \qquad v^* = x^{*-1/4} g$$

So:
$$x^{*1/2} f \left(-\frac{1}{4} \frac{\zeta}{x^*} h' \right) + x^{*-1/4} g \, x^{*-1/4} h' = x^{*-1/2} h''$$

$$\boxed{\therefore\ h'' = g h' - \frac{1}{4} \zeta f h'}$$

$$\boxed{\begin{array}{c} h'' = h' \left(g - \frac{1}{4} \zeta f \right) \\ h(0) = 1 \qquad h(\infty) = 0 \end{array}}$$

And momentum:

$$\frac{\partial^2 u^*}{\partial y^{*2}} = f'' \qquad \frac{\partial u^*}{\partial y^*} = x^{*1/4} f'$$

So
$$\frac{1}{Pr} \left(x^{*1/2} f \, \frac{1}{2} x^{*-1/2} (f - \zeta f') + x^{*-1/4} g \, x^{*1/4} f' \right)$$
$$= f'' + h$$

$$f'' = -h + \frac{1}{Pr}\left(\frac{1}{2}\left(f^2 - 3ff'\right) + g f'\right)$$

$$f(0) = 0 \qquad f(\infty) = 0$$

Unknowns are $f'(0)$, $h'(0)$ (e.g., wall shear stress and heat flux)

Let $$\underset{\sim}{y} = \begin{bmatrix} f \\ f' \\ g \\ h \\ h' \end{bmatrix} = \begin{bmatrix} y_1 \\ y_2 \\ y_3 \\ y_4 \\ y_5 \end{bmatrix}$$

$$\therefore \underset{\sim}{y}' = \begin{bmatrix} y_2 \\ -y_4 + \frac{1}{Pr}\left(\frac{1}{2}\left(y_1^2 - 3y_1y_2\right) + y_3y_2\right) \\ -\frac{1}{2}\left(y_1 - 3y_2\right) \\ y_5 \\ y_5\left(y_3 - \frac{1}{4}3y_1\right) \end{bmatrix}$$

$$\text{IC}: \begin{bmatrix} 0 \\ x_1 \\ 0 \\ 1 \\ x_2 \end{bmatrix}$$ for two shooting param.

minimize norm(y_1, y_4) as $\eta \to \infty$

```
function out=miss8(x)
% This function takes in the unknown wall shear stress and heat flux and
% returns zero if the boundary conditions at infinity are satisfied.
% Because the integral is unstable for large eta unless the correct initial
% conditions are specified, we "sneak up" on the solution.

% call function with:
%
% global etalim;etalim=2;x=[1;-1];
%
% for i=1:5;x=fminsearch('miss8',x);etalim=etalim*1.5;end;x

global etalim

y0 = [0 x(1) 0 1 x(2)]'; %The initial condition

pr = .7; %The prandtl number

ydot = @(eta,y) [y(2);-y(4)+1/pr*((y(1)^2-eta*y(1)*y(2))/2+y(3)*y(2));...
    -(y(1)-eta*y(2))/2;y(5);y(5)*(y(3)-eta*y(1)/4)];

[etaout yout] = ode23(ydot,[0 etalim],y0);

figure(1)
plot(etaout,yout)
xlabel('eta')
ylabel('function values')
legend('f','fp','g','h','hp')
grid on
drawnow

out = norm([yout(end,1),yout(end,4)]);
```