P.O.D 10

Review of Solution Techniques

1) Write down eq'n & BC's and scale!
   - ⇒ Usually - subtract off ref. temp. to start
   - ⇒ Length scale - usually - det. by BC's
   - ⇒ Time scale - usually - det by balance of time deriv & cond term in DE
   - ⇒ Temp scale - usually - det. by inhomogeneity driving problem (source in DE, BC at wall, IC)

Scaling generally tells you a lot about what is going on!

2) Solve for asymptotic solution at long times
   - ⇒ Usually - this is steady, but not always!
   - ⇒ Sol'n to ODE depends on problem!

⇒ think about energy flow - where is it coming from, where is it going?

⇒ If BC's are all fixed heat flux, asymptotic solution is often unsteady - no where for energy to go, so it piles up!

⇒ In this case (& if linear) then -often- $T_\infty \sim t\, f_1(x) + f_2(x)$, etc.

⇒ If asymptotic DE is linear you can usually solve it, if non-linear usually do it numerically w/ ODE solver, often w/ shooting method.

3) If problem is linear, subtract off asymptotic solution to get a separable PDE w/ homogeneous BC's & inhomogeneous IC

4) Spatial part is -usually- a SL problem

⇒ solve analyically if you can!

⇒ solve numerically if you can't!

⇒ determine lead eigenvalue!!

⇒ get everything else too! (But lead eigenvalue is most important)

5) If scaling is length scale deficient (or if you just think it might work)

⇒ Use affine stretching to see if there is "a free wish left over".

⇒ Put restrictions in canonical form

⇒ Get similarity rule & variable!

⇒ Get transformed ODE & solve ~~numer~~ analytically or numerically

⇒ Can work for both linear & non-linear problems, and great when it does!

6) For periodic problems...

⇒ Usually-only- the asymptotic sol'n matters, but it's unsteady!

⇒ If it's linear try analytic continuation in complex plane

7) If problem is non-linear (and doesn't admit self-similar sol'n!), usually solve it numerically

⇒ If (fairly) simple parabolic PDE, try marching solution (watch for instability!)

⇒ If more complicated eq'n, geometry, etc., use a canned finite element solver.

These methods have been introduced for heat transfer in solids, but we will use the same techniques for convective flows, mass transfer, momentum transfer, etc.!

Only the application (& governing eq'n & BC's) change!

Key concepts

1) thermal conductivity : $\frac{\text{Energy}}{\text{Length} \cdot \text{time} \cdot {}^\circ K}$

⇒ high for metals, low for gases!

2) thermal diffusivity : $[\alpha] \equiv \frac{\text{Length}^2}{\text{time}}$

$\alpha = \frac{k}{\rho \hat{C}_p}$ — diffusion coef. for energy.

⇒ fairly high for (some) metals due to high conductivity

⇒ also high for gases due to low $\rho \hat{C}_p$

⇒ lower for liquids like water!

3) Diffusion length and time :

$\delta \sim (\alpha t)^{1/2}$ $\quad t_c \sim b^2/\alpha$

4) Heat transfer coefficient ⇒ $\frac{\text{Power}}{\text{Area} \cdot {}^\circ K}$

$h \sim \frac{k}{D}$ ← appropriate length

$q = h \Delta T$!

5) Biot Number: $\dfrac{hR}{k}$ ← external ht transf coef; k ← internal conductivity

$\equiv \dfrac{\text{internal resistance}}{\text{external resistance}} = \dfrac{h}{(k/R)}$

If $Bi \ll 1$ internal temp is usually pretty uniform!

— can use lumped capacitance model!

6.) Nusselt Number $Nu = \dfrac{hD}{k}$ ← external conductivity! (for ext. ht transf)

basically, it's a dimensionless heat transfer coefficient! $= \dfrac{h}{k/D}$

(ht transf coef divided by that for steady ht transfer throug a slab of thickness D)

7) $Re = \dfrac{UD}{\nu} = \dfrac{\text{Inertia}}{\text{viscous}}$

8) $Pr = \nu/\alpha = \dfrac{\text{mom dof}}{\text{thermal dof}}$

9) $RePr \equiv$ Peclet Number $= \frac{UD}{\alpha}$

This is $\frac{\text{convection}}{\text{thermal diffn}}$ – we'll be working with this a lot next week!