Lecture 8 Demo: Using a lightbulb filament as a hot wire anemometer

Filament characteristics:

old style incadescent lightbulb from miniature christmas lights

Filament is a tungsten coil w/ diameter of ~ 20μm (wire) and ~100μm (coil). wire length (stretched out) is about 2cm, coil length is about 2mm

A lot of variability between filaments!

Properties:

$R_0 = 4\,\Omega$, $\lambda = 0.0045\ {}^1/{}_{°C}$ temp. coef

$K = 0.026\ \frac{W}{m\,°K}$ for air

$\nu = 0.15\ \mathrm{cm^2/s}$

We can operate in constant current or constant voltage mode.

Look at constant current mode first!

We have the energy balance:

$$I^2 R = \pi D L h \Delta T$$

Work w/ Nusselt number $Nu = \frac{hD}{k}$

$$\therefore I^2 R_0 (1 + \lambda \Delta T) = \pi L k \, Nu \, \Delta T$$

or $$1 + \lambda \Delta T = \left(\frac{\pi L k \, Nu}{I^2 R_0 \lambda} \right) \lambda \Delta T$$

$$\therefore \quad \lambda \Delta T = \frac{1}{\left(\frac{\pi L k \, Nu}{I^2 R_0 \lambda} \right) - 1}$$

This points out the hazards of constant current mode: If $\frac{\pi L k \, Nu}{I^2 R_0 \lambda} \leq 1$ your wire fries!

We can estimate the critical Nu for a particular current.

$$Nu_{cr} = \frac{I^2 R_0 \lambda}{\pi L k}$$

for our wire we have issues with

$I \gtrsim 0.2\,A$

$$\text{so } Nu_{cr} \approx \frac{(0.2)^2(4)(4.5\times10^{-3})}{\pi(2\times10^{-3})(0.026)} = 4.4$$

This is very approximate as the length and other wire properties aren't very consistent!

What sort of Nu would we expect?

From Whittaker:

$$Nu = \left(0.4\,Re^{1/2} + 0.06\,Re^{2/3}\right)Pr^{0.4}$$ (ignore μ correction)

for air $Pr = 0.7$

so $Nu \approx 0.4\,Re^{1/2}$ to a good approx.

$$Re = \frac{UD}{\nu}$$

If $U = 1\,m/s = 100\,cm/s$ and $D = 0.01\,cm$ (coil D)

then $Re \approx 7$ (pretty low!)

This would yield $Nu \approx 1$

which is too low!

Instead let's work with the total length of the uncoiled wire of 2cm, 20μm

$\therefore \; Nu_{cr} \approx 0.44$

but $Re = \dfrac{(100)(0.002)}{0.15} = 1.3$

which would yield $Nu \approx 0.46$

This now matches, but it is still quite uncertain!

suppose we <u>measure</u> some voltage V.

What is the corresponding Nu?

$$V = I\Omega = IR_0(1 + \lambda\Delta T)$$

$$\therefore \; \lambda\Delta T = \frac{V}{IR_0} - 1$$

$$\frac{\pi L k Nu}{I^2 R_0 \lambda} - 1 = \frac{1}{\frac{V}{IR_0} - 1}$$

$$\text{So} \quad Nu = \frac{\lambda I^2 R_0}{\pi L k}\left(1 + \frac{1}{\frac{V}{IR_0} - 1}\right)$$

$$= \frac{\lambda I^2 R_0}{\pi L k}\left(\frac{\frac{V}{IR_0}}{\frac{V}{IR_0} - 1}\right)$$

$$= \frac{\lambda I^2 R_0}{\pi L k}\left(\frac{V}{V - V_0}\right) \qquad \text{where } V_0 = IR_0$$

$$\text{so} \quad U \approx \left(\frac{\nu}{(0.4)^2 D}\right)\left[\frac{\lambda I^2 R_0}{\pi L k}\left(\frac{V}{V - V_0}\right)\right]^2$$

For a real system you would generate a calibration curve, but it should be similar. Note that the min velocity you could measure is where $V/V_0 \to \infty$, yielding

$$U_{min} = \left(\frac{\nu}{(0.4)^2 D}\right)\left(\frac{\lambda I^2 R_0}{\pi L k}\right)^2$$

This would be the frying speed!

Now for constant voltage

$$\frac{V^2}{R} = \frac{V^2}{R_0(1+\lambda\Delta T)} = \pi L k Nu \Delta T$$

or $$\frac{V^2\lambda}{R_0 \pi L k Nu} = (1+\lambda\Delta T)(\lambda\Delta T)$$

Thus $$\lambda\Delta T = \frac{-1+\sqrt{1+4\dfrac{V^2\lambda}{R_0\pi L k Nu}}}{2}$$

Let's let $\dfrac{V^2\lambda}{R_0\pi L k Nu} \equiv \chi$

$$\therefore \lambda\Delta T = \frac{1}{2}\left((1+4\chi)^{1/2}-1\right)$$

This actually behaves for large χ:

As $4\chi \gg 1$ $\quad \lambda\Delta T \sim \chi^{1/2}$

so you can protect your filament better - no runaway occurs (although very large χ may still yield too large a ΔT!

In this case we measure some current I

s.t. $I = \frac{V}{R} = \frac{V}{R_o(1+\lambda\Delta T)}$

$\therefore \lambda\Delta T = \frac{V}{R_o I} - 1 \equiv \frac{I_o}{I} - 1$

So $\frac{V^2\lambda}{R_o \pi L k Nu} = \frac{I_o}{I}\left(\frac{I_o}{I} - 1\right)$

or $Nu = \frac{V^2\lambda}{R_o \pi L k} \frac{I}{I_o}\left(\frac{I_o}{I} - 1\right)^{-1}$

$= \frac{\lambda V}{\pi L k} \frac{I}{\frac{I_o}{I} - 1} = \frac{\lambda V I}{\pi L k}\left(\frac{I}{I_o - I}\right)$

which again could be used to calculate a velocity!

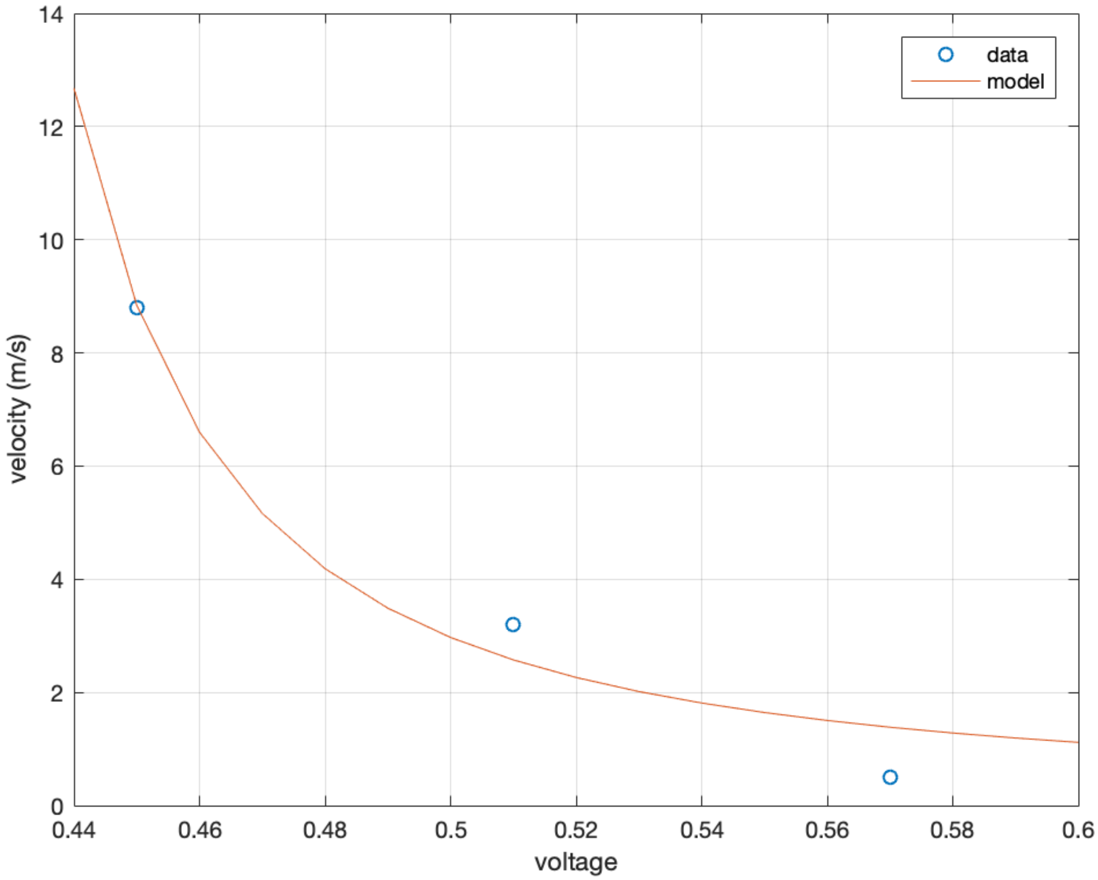
Contents

Analysis of Heated Wire Anemometer Demo

In this script we analyze the data for the heated wire anemometer demonstration. The data is really uncertain, however we were able to get three voltages at three different wind speeds for a constant current on our lightbulb filament. From the analyis we expect that, at least over a certain range, the velocity should go as umin * (v/(v-v0))^2 (e.g., assuming that the Nusselt number goes as Reynolds number to the 1/2 power). We can do a non-linear curve fit to the data to get the fitting parameters:

```
v = [.45 .51 .57]'; % measured voltage
u = [8.8 3.2 0.5]'; % measured wind in m/s

% We define a "miss" function:

miss = @(x) norm(u - x(1)*(v./(v-x(2))).^2);

x = fminsearch(miss,[.3 .4]);

umin = x(1)
v0 = x(2)

vp = [.44:.01:.6]'; % an array of voltages for plotting purposes

figure(1)
plot(v,u,'o',vp,x(1)*(vp./(vp-x(2))).^2)
grid on
xlabel('voltage')
ylabel('velocity (m/s)')
legend('data','model')
```

```
umin =

    0.1298


v0 =

    0.3955
```



Results

The calibration curve actually isn't too bad - but that is largely fortuitous. Because of the way the objective function (e.g., 'miss') is chosen it is dominated by the measurement at the highest wind speed. This isn't necessarily bad, however, as those conditions are a bit more reproducible. At low wind speeds you wouldn't have Nu proportional to U^.5 anyway, because natural convection would come into play too! The frying speed umin = 0.13 m/s can be compared to what would be expected for such a filament. If we use the fitted value of V0 to get R0, take the filament to be 2mm long and 100 microns in diameter then we get a "frying speed" of 2.5 m/s, quite a bit higher than the fitted value of .13 m/s. This isn't terribly surprising as the filament dimensions are very uncertain! The calibration curve would be a better approximation of reality at the higher wind speeds in any event.

It is interesting to note that there is a pretty large temperature change between the lowest and highest wind speeds. Based on the change in resistance, the temperature at the lowest speed is about 60°C higher than it was at the highest wind speed.

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