POD Lecture 07

• Quenching sphere w/ Biot #

$$T\Big|_{t=0} = T_1 \qquad q_r\Big|_{r=a} = h\left(T\Big|_{r=a} - T_0\right) = -k\frac{\partial T}{\partial r}\Big|_{r=a}$$

$$\therefore \quad \rho \hat{C}_p \frac{\partial T}{\partial t} = k\frac{1}{r^2}\frac{\partial}{\partial r}\left(r^2 \frac{\partial T}{\partial r}\right)$$

$$T^* = \frac{T - T_0}{T_1 - T_0} \qquad t^* = \frac{t k}{\rho \hat{C}_p a^2} = \frac{\alpha t}{a^2} \qquad r^* = r/a$$

$$\therefore \quad h T^*\Big|_{r^*=1} = -\frac{k}{a}\frac{\partial T^*}{\partial r^*}\Big|_{r^*=1}$$

$$\text{or} \quad T^*\Big|_{r^*=1} + \underbrace{\frac{k}{ah}}_{\frac{1}{Bi}}\frac{\partial T^*}{\partial r^*}\Big|_{r^*=1} = 0$$

$$\frac{\partial T^*}{\partial r^*}\Big|_{r^*=0} = 0 \quad \text{(symmetry, finite value)}$$

$$\text{So:} \quad \frac{\partial T^*}{\partial t^*} = \frac{1}{r^{*2}}\frac{\partial}{\partial r^*}\left(r^{*2}\frac{\partial T^*}{\partial r^*}\right)$$

$$T^* = F(r^*)G(t^*)$$

$$\therefore \quad \frac{G'}{G} = \frac{(r^{*2}F')'}{r^{*2}F} = -\lambda \quad ; \quad G = e^{-\lambda t^*}$$

$$(r^{*2}F')' - \lambda r^{*2}F = 0$$

$$F'(0) = 0 \qquad F(1) + \frac{1}{Bi}F'(1) = 0$$

$$\text{So } T^* = \sum_{n=1}^{\infty} A_n e^{-\lambda_n t^*} F_n(r^*)$$

we seek $T^*\Big|_{r^*=0}$ as $f^{\underline{n}}(t^*)$

This <u>has</u> an analytic solution, but it is a bit messy! (many such sol. are found in Carslaw & Jaeger – nice compendium)

Do it numerically instead!