Problem of the Day Lecture 05

Internal heat transfer coef. for a quenched sphere

Often we want to look at transient problems in terms of an "average" heat transfer coef.

Suppose we have some average temp. of a sphere $\bar{T}$. We can define

$$q_r\Big|_{r=1} = h\left(\bar{T} - T\Big|_{r=a}\right)$$

We can get h from the SL solution!

After the initial transient (for $Bi \to \infty$)

$$T^* = 2e^{-\pi^2 t^*} \frac{\sin \pi r^*}{\pi r^*}$$

What is $\bar{T}$?

$$\bar{T}^* = \frac{1}{\frac{4}{3}\pi} \int_0^1 T^* 4\pi r^{*2} dr^* = \frac{6}{\pi} e^{-\pi^2 t^*} \int_0^1 r^* \sin \pi r^* dr^*$$

$$= \frac{6}{\pi^2} e^{-\pi^2 t^*}$$

Note that this is less than 1 (for coef) because higher eigenvalues decay faster! (lose heat from larger r^*)

We also have the heat flux:

$$q_r = -k\frac{\partial T}{\partial r}\Big|_{r=a} = -\frac{k}{a}(T_1 - T_0)\frac{\partial T^*}{\partial r^*}\Big|_{r^*=1}$$

$$= -\frac{k}{a}(T_1 - T_0)\frac{2}{\pi}\underbrace{\frac{\partial}{\partial r^*}\left(\frac{\sin \pi r^*}{r^*}\right)\Big|_{r^*=1}}_{(-\pi)} e^{-\pi^2 t^*}$$

$$= 2\frac{k}{a}(T_1 - T_0)\, e^{-\pi^2 t^*}$$

$$h = \frac{q_r}{\bar{T} - T_0} = \frac{2\frac{k}{a}\cancel{(T_1 - T_0)}\,\cancel{e^{-\pi^2 t^*}}}{\cancel{(T_1 - T_0)}\frac{6}{\pi^2}\cancel{e^{-\pi^2 t^*}}}$$

$$= \frac{\pi^2}{3}\frac{k}{a}$$ after initial transients from higher eigenvalues have dissipated.

Coef is greater than 1 (e.g., not just k/a)

because most of ht energy is close to the outer edge!

How could you use this? Heat a bucket of water (well stirred) w/ a hot rock.

Fraction of energy released fast is

$1 - \frac{6}{\pi^2} \approx 0.39$ of total energy

This is released in time ~ $\frac{a^2}{\alpha} \frac{1}{(2\pi)^2}$ 2nd eigenvalue

Thereafter your exchange is governed by this effective ht transfer coef!