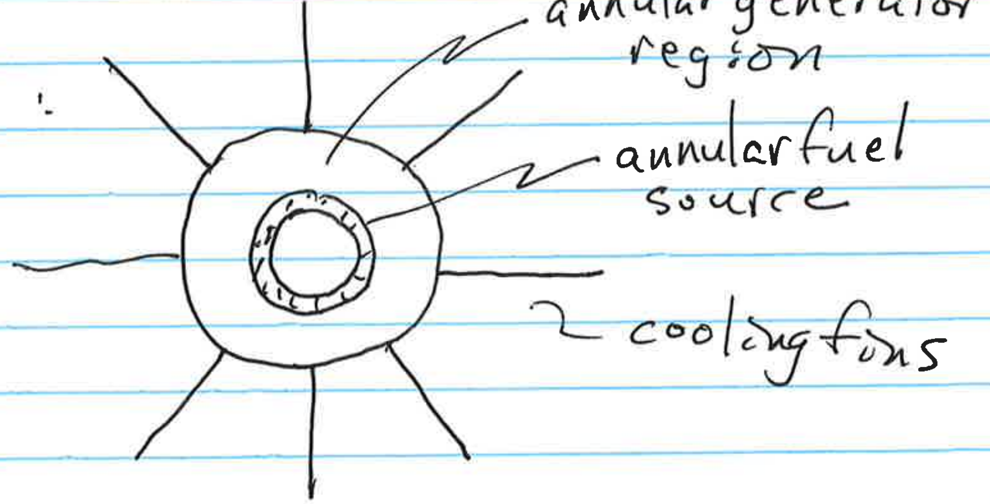
Lecture 03 P.O.D.

What are the heat transfer characteristics of the SNAP-27 RTGs used in the lunar missions?

RTG: Radionucleid Thermoelectric Generator

- Uses thermoelectric (seebeck) effect to directly convert a temp. gradient into electricity
- Not very efficient (about 10% max) but no moving parts!
- Uses radionucleid (Pu238 or other α emitters - low shielding requirement) to produce heat, + sink to produce gradient
- design:



3 Questoons:

1) What is the "effective temp." of the radiators?

2) What is the effective thermal conductivity of the annular RTG?

3) What is the temperature of the central vood?

Some numbers:

$L = 35$ cm (fuel length) 41.9 cm (cannister length)

$D = 6.4$ cm (cannister diameter)

3735 g fuel, 10.3 g/cm^3 particle density (~~[illegible]~~)

2.6 W/cm^3 power density

~~[illegible] power density~~ 0.40 W/g

1480 W thermal energy volume = 569 cm^3

(packing fraction $= \left(\frac{3735}{569}\right)/10.3 = \frac{6.56}{10.3} = 0.64$)

$$k_{fuel} = 0.62 \frac{Btu}{hr\,ft\,^\circ F} = 1.07 \frac{W}{m\,^\circ K}$$

If the volume is 569 cm^3, length is 35 cm, OD is 6.4 cm, what is ID?

$$\pi \left(\frac{OD}{2}\right)^2 - \pi \left(\frac{ID}{2}\right)^2 = \frac{569}{35}$$

$$OD^2 - ID^2 = \frac{4}{\pi}\frac{569}{35} = 20.7\,cm^2$$

$$\therefore ID = 4.5\,cm$$

So thickness is just 0.95 cm (pretty thin)

The RTG layer has an inner diameter of 6.4 cm. From the picture, the outer diameter is about 3x this, or 19.2 cm

Thus for RTG, $R_1/R_0 \approx 3$

For a cylindrical geometry:

$$\frac{Q}{2\pi L}\frac{\ln(R_1/R_0)}{K} = T_1 - T_0$$

From the report, $Q = 1480$ W

$T_1 = 866\,°K$ $\quad T_0 = 547\,°K$ $\quad \therefore T_1 - T_0 = 319\,°K$

$$\therefore k_{RTG} = \frac{1480\,W}{2\pi\,(0.42\,m)}\,\frac{\ln(3)}{319\,°K} = 1.93\,\frac{W}{m\,°K}$$

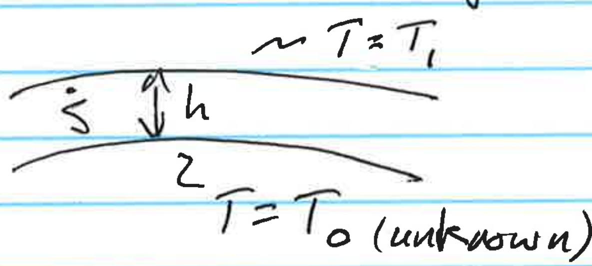
This is about the thermal conductivity of porcelain (a bit higher)

It is the conductivity of silicon-germanium semiconductors often used in RTGs. ($1-2\,\frac{W}{m\,°K}$)

How much hotter is the inside of the fuel layer than the outside?

- thickness = 0.95 cm << 3.2 cm outer radius

- approximate w/ a slab (flat earth limit) otherwise would get ~~Bessel functions~~ logarithm terms!



$$0 = k\frac{\partial^2 T}{\partial y^2} + \dot{S}$$

$$y^* = y/h\ ,\quad T^* = \frac{(T - T_1)}{\Delta T_c}$$

$$T^*\Big|_{y^*=1} = 0 \quad \frac{\partial T^*}{\partial y^*}\Big|_{y^*=0} = 0 \text{ (insulated void)}$$

$$\frac{k\,\Delta T_c}{h^2}\frac{\partial^2 T^*}{\partial y^{*2}} = -\dot{S}$$

$$\frac{\partial^2 T^*}{\partial y^{*2}} = -\boxed{\frac{\dot{S} h^2}{k\,\Delta T_c}}$$

$$\quad\quad\quad\quad\quad\quad\quad\quad\quad\quad \shortparallel$$

$$\quad\quad\quad\quad\quad\quad\quad\quad\quad\quad 1$$

so $\Delta T_c = \dfrac{\dot{S} h^2}{k}$

$\dot{S} = 2.6\ \dfrac{W}{cm^3}$ $\quad h = 0.95\,cm$ $\quad k = 1.07\ \dfrac{W}{m\,^\circ K} = 0.01\ \dfrac{W}{cm\,^\circ K}$

$$\therefore \Delta T_c = \frac{(2.6)(0.95)^2}{0.0107} = 219\ ^\circ K$$

What is $T^*\Big|_{y^*=0}$?

$$T^* = -\tfrac{1}{2}y^{*2} + Ay^* + B$$

$$\frac{\partial T^*}{\partial y^*}\Big|_{y^*=0} = 0 \ \therefore A = 0 \qquad T^*\Big|_{y^*=1} = 0 \ \therefore B = \tfrac{1}{2}$$

so $T^*\Big|_{y^*=0} = \frac{1}{2}$ and the inside is 110°K hotter than outside!

Finally, look at radiative cooling

$$q = \sigma T^4$$

$\sigma \to$ Stefan-Boltzmann Constant

$$\sigma = 5.67 \times 10^{-8} \frac{W}{m^2 \, {}^\circ K}$$

The outside is $547^\circ K$

If we have a length of 0.4 m and

$$q = \frac{Q}{(L)\pi D} = \frac{1480 - 63.5\,W}{(0.4\,m)\pi D} = 5.67 \times 10^{-8} (547)^4$$

(63.5 W was removed by production)

$$\therefore D = \frac{1416}{(0.4)(\pi)(5.67 \times 10^{-8})(547)^4} = 0.22\,m$$

or 22 cm

This is a bit small (more like ~ 50 cm from picture) because it ignores back radiation and alot of the fins are at a lower temperature. Also, the emissivity $\varepsilon < 1$.

We'll look at fins next time!