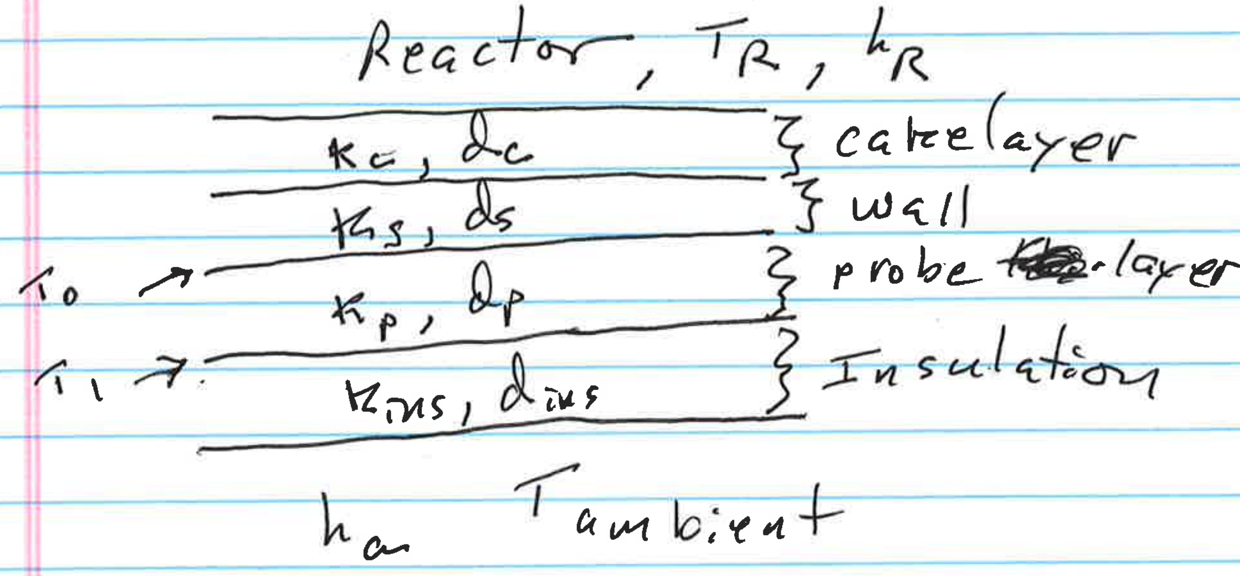
Problem of the Day : Lecture 2

- Idaho Waste treatment facility a reactor has a tendency to form cake layer on the wall. It is necessary to "catch this early" to avoid serious issues. A proposed technique is to use a thermal probe on the outside of the reactor to estimate cake layer thickness. Determine the specs and reliability of such a probe.



Using knowledge of T_0, T_1 we want to calculate δ_c!

Solution:

In this geometry q is constant!

we can measure this from T_0 & T_1:

$$q = (T_0 - T_1)\frac{k_p}{d_p}$$

we also have the flux through the interior layers:

$$q = (T_R - T_0)\frac{1}{\frac{1}{h_R} + \frac{d_c}{k_c} + \frac{d_s}{k_s}}$$

so:
$$(T_0 - T_1)\frac{k_p}{d_p} = (T_R - T_0)\frac{1}{\frac{1}{h_R} + \frac{d_c}{k_c} + \frac{d_s}{k_s}}$$

$$\frac{1}{h_R} + \frac{d_c}{k_c} + \frac{d_s}{k_s} = \frac{d_p}{k_p}\frac{(T_R - T_0)}{(T_0 - T_1)}$$

$$\therefore d_c = k_c\left[\frac{d_p}{k_p}\frac{(T_R - T_0)}{(T_0 - T_1)} - \frac{1}{h_R} - \frac{d_s}{k_s}\right]$$

Note that you don't *need* to know anything about the insulating layer or external h_a to calculate δ_c, *but* the insulation will *increase* error!

To get a good measurement, you want things to be *dominated* by δ_p / k_p *and* δ_c / k_c as much as practical!

If h_a and $\left(\frac{k_{ins}}{\delta_{ins}}\right)^{-1}$ are too small, then the overall heat flux will be small and $T_1 - T_0$ will be too small to measure accurately.

Let's pick some numbers!

$$h_R = \infty$$

$$h \cong \frac{k}{D}\left(0.5\left(\frac{UD}{\alpha}\right)^{1/2}\right)$$

$$\cong 0.5\left(\frac{U\, k\, \rho C_p}{D}\right)^{1/2}$$

$U = 4\ m/s$, $k = 0.02\ W/m\,^{\circ}K$

$D = 2\ m$ $\rho = 1.2\ kg/m^3$

$C_p = 10^3\ \frac{J}{kg\,^{\circ}K}$

$$\therefore\ h_a \cong 0.5\left(\frac{(4\ m/s)(0.02\ \frac{W}{m\,^{\circ}K})(1.2\ \frac{kg}{m^3})(10^3\ \frac{J}{kg\,^{\circ}K})}{2\ m}\right)^{1/2}$$

$$\cong 3.5\ \frac{W}{m^2\,^{\circ}K}$$

$k_{ins} \cong 0.035\ \frac{W}{m\,^{\circ}K}$ $d_{ins} = 0.05\ m$ (2")

$\therefore\ h_{ins} = \frac{k}{d} = 0.7\ W/m^2\,^{\circ}K$

$k_{steel} = 45\ \frac{W}{m\,^{\circ}K}$ $d_s = 0.0127\ m$ (1/2" steel)

$\therefore\ h_s = 3{,}543\ \frac{W}{m^2\,^{\circ}K}$ (very high

Take probe to have thickness 1 cm = 0.01 m & insulating layer w/ $k_p \approx 0.035 \frac{W}{m^\circ K}$

$\therefore \; h_p = 3.5 \frac{W}{m^2 {}^\circ K}$

$k_{cake} = 0.2 \frac{W}{m^\circ K}$

want to measure layer which is 2 cm = 0.02 m thick. $\therefore \; h_c \sim 10 \frac{W}{m^2 {}^\circ K}$

$T_R = 400^\circ C \quad T_a = 30^\circ C$

what is q?

$$q = (T_R - T_a) \frac{1}{\cancel{\frac{1}{h_R}} + \frac{1}{h_c} + \frac{1}{h_s} + \frac{1}{h_p} + \frac{1}{h_{ins}} + \frac{1}{h_a}}$$

Ignore

$$= (370)\left(\frac{1}{10} + \frac{1}{3543} + \frac{1}{3.5} + \frac{1}{0.7} + \frac{1}{3.5}\right)^{-1}$$

$$= 176 \; W/m^2$$

what is $T_R - T_0$?

$$T_R - T_0 = \frac{q}{\left(\frac{1}{h_R} + \frac{1}{h_C} + \frac{1}{h_S}\right)^{-1}} = 17.6^\circ$$

$$T_0 - T_1 = \frac{q}{\left(k_p / d_p\right)} = 50.3^\circ$$

So you could get away with these numbers – and even measure thinner layers...