CBE 30355 TRANSPORT PHENOMENA I

Mid-Term Exam
10/17/24

This test is closed books and closed notes

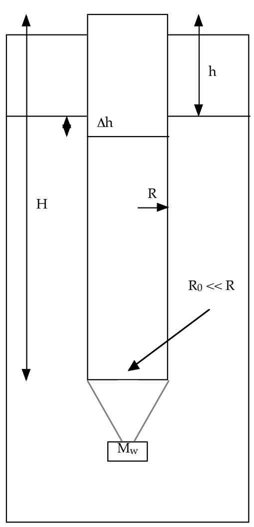
Problem 1 (20 points). Hydrostatics/Archimedes Law/Bernoulli's Equation. Time keeping has played an important role in human civilization, and many techniques have been developed over the millennia. An early approach was the Persian Water Clock, in which a bowl with a small hole in the bottom was floated in a reservoir. When the bowl filled sufficiently it abruptly sank, with the sinking time providing one measured unit of time (an attendant would then refill it and keep track of the number of times it was repeated). Here we analyze such a system.

Consider the massless cylindrical can of radius R and height H depicted below (in this case a tennis ball can). A small hole of radius R_0 is drilled into the bottom of the can, and a mass $M_w = \rho_w V_w$ of density ρ_w and volume V_w is attached to the bottom. Because of the weight, the water level in the can is a height Δh below the level in the surrounding reservoir (in this case a glass vase). This causes water (density ρ) to flow into the can through the hole in the bottom. The top of the can projects a distance h above the reservoir level, and this will decrease as the can fills.

a. How does the inflow velocity U depend on the parameters of the problem? Assume that there are no losses in the system.

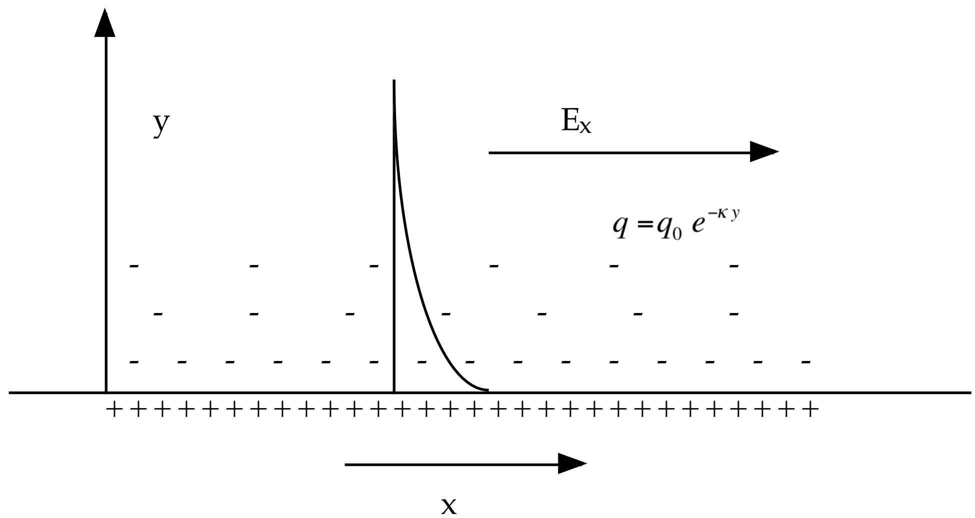
b. Write down a differential equation and boundary conditions for how the projecting height h varies with time.

c. Solve the problem to determine the sinking time. Note that this works quite well, although the time needs to be adjusted by the inverse of the discharge coefficient (about 0.6) to account for frictional losses in the hole.



Problem 2 (20 points). Unidirectional Flow. A technique used to drive fluid through microchannels is *electroosmosis*, in which fluid (usually water) is driven via tangential electric fields. Here we look at this problem and calculate a quantity known as the electroosmotic mobility.

Consider the surface depicted below. In an aqueous system ionic species are often adsorbed onto the surface (these are fixed charges), and counter-ions are distributed in a very thin diffuse layer above the surface (this occurs due to diffusion). The net free charge density q decays away exponentially away from the surface, e.g.,



where q_0 is the charge density at y = 0 and the constant κ^{-1} is known as the Debye length (usually just a few microns or less!). If you have a tangential electric field E_x (e.g., in the x direction), you get a body force (just like gravity!) in the x direction of magnitude $E_x q$ that dies away exponentially far from the surface. The boundary conditions are no-slip at the surface y = 0 and zero shear stress far away (which because the Debye length is so short isn't very far at all!). Using this:

a. Write down the equation governing the velocity profile in the x-direction including this body force.

b. Render the equation dimensionless and solve it to get the velocity profile.

c. The electroosmotic mobility is just the velocity far away divided by the electric field strength. What is it?

Problem 3 (10 points). Unidirectional Flow/Scaling. In class we demonstrated how you could measure (if not terribly accurately) the viscosity of corn syrup by looking at flow down an incline in a shallow tray. The corn syrup was poured into the tray to a depth d, a marker was placed on the surface of the fluid, and then at time $t = 0$ the tray was tilted to an angle θ relative to the horizontal. The velocity of the marker was measured and used to calculate the kinematic viscosity. The question here is how long we needed to wait for the velocity of the marker to achieve steady state and what kind of fluid we can use this approach for.

a. Write down the governing equations and boundary conditions for this unidirectional flow problem for a tray at angle θ. Don't forget the transient term!

b. Render the equations dimensionless, determining the characteristic velocity (use steady-state here, as that is what we were trying to measure!) and characteristic time.

c. How long would we have to wait to ignore the transient term? If the tray is of length L, what would be the approximate maximum depth or minimum viscosity we could get away with? (Hint: just combine U_c and t_c and compare to L…)

Problem 4 (10 points). Conservation of Momentum. Dinosaur killing asteroid deflection has been much in the news lately. While there are many options if you find it early enough, if is picked up too late you basically have to use nuclear weapons to give you enough change in momentum to do any good (and even that's hard). Suppose the energy yield of your weapon is E. You have a couple of options for how you can use it: You can blow it up on the surface, giving a small mass of ejected material M_e a very high velocity, or you can blow it up deeper in (e.g., ground penetrating charge, or land and bury it) and eject a larger mass with a lower velocity. In this problem you are going to examine these options.

a. Draw a sketch of your system surrounded by a control volume.

b. The total momentum of the system is conserved. If all the energy of the explosion is converted to the kinetic energy of the ejected material, and if it is all ejected in the same direction, how does the change in velocity of the asteroid of mass $M_a >> M_e$ depend on the parameters? (Note: we will ignore the gravitational attraction of the asteroid, assuming that the ejected velocity is much greater than the escape velocity – this would actually place a constraint on M_e.)

c. Based on your expressions, what is the better way to use your nuke?