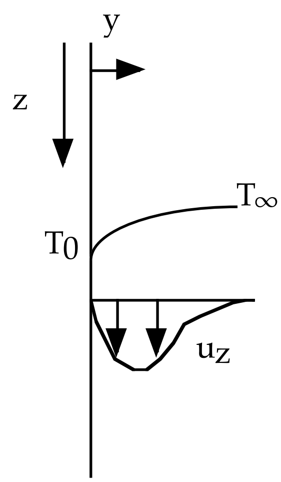
CBE 30355 Transport Phenomena I
Final Exam

December 16 & 18, 2024

Closed Books and Notes

Problem 1. (20 points) Thermal and Momentum boundary layers. It's cold outside and that means drafts off of cold windows! The cause is straightforward: the density of air increases as it gets colder according to the ideal gas law, and that causes a negative buoyant force that drags the fluid down. In this problem you will scale the transport equations to determine characteristic magnitude of the heat loss from a cold window from scaling alone.

Consider the window of height H depicted below. We take the temperature of the air in the room to be T_∞ and that of the window to be T_0, so the characteristic temperature difference is just $\Delta T = -(T_0 - T_\infty)$. The Navier-Stokes equation for momentum in the z direction reduces to:



$$\left(u_y \frac{\partial u_z}{\partial y} + u_z \frac{\partial u_z}{\partial z}\right) = \nu\left(\frac{\partial^2 u_z}{\partial y^2} + \frac{\partial^2 u_z}{\partial z^2}\right) - \beta g\left(T - T_\infty\right)$$

$$\frac{\partial u_y}{\partial y} + \frac{\partial u_z}{\partial z} = 0$$

$$Q/W = \int_0^H -k \left.\frac{\partial T}{\partial y}\right|_{y=0} dz$$

For natural convection problems such as this, the pressure gradient (a source of momentum!) is replaced with the buoyancy term $\rho g \beta (T - T_\infty)$ where β is the coefficient of thermal expansion (just $1/T_\infty$ for an ideal gas, where T_∞ is in °K). The rate of heat loss per unit width Q/W is given by the integral of the temperature gradient at the surface, where k is the thermal conductivity of the air. Scaling these equations (and the continuity equation!) you can determine the boundary layer thickness, the characteristic magnitude of the downward draft velocity, and an estimate of the rate of heat loss from the window without ever solving the problem!

a. Render the z-momentum and continuity equations dimensionless in the boundary layer limit.

b. Determine how the boundary layer thickness and draft velocity depend on the parameters of the problem.

c. How large does H need to be for the boundary layer limit to be valid?

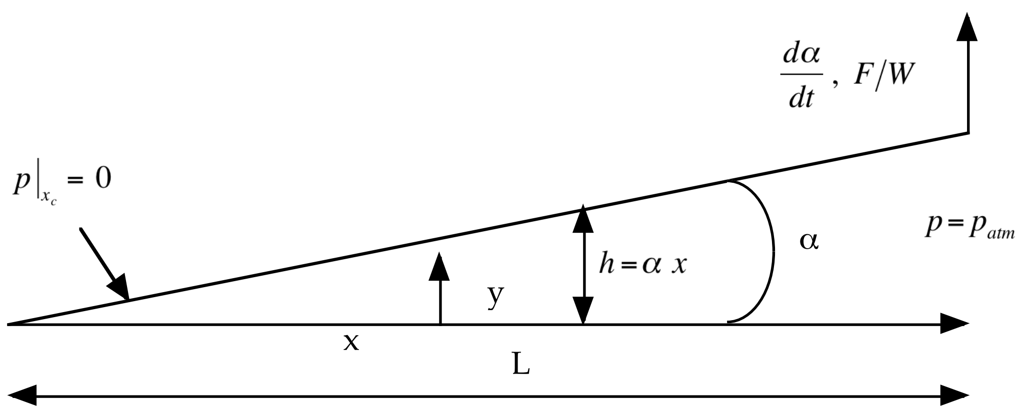
d. In the boundary layer limit the problem admits a self-similar solution (it's not a function of y^* and z^* independently, but rather a function of the similarity variable η). From your scaling (or via simple affine stretching – same thing!) determine η in canonical form.

e. By scaling the integral relation for the heat loss from the window, determine how Q/W depends on the parameters of the problem.

f. Now for some numbers. If ΔT is 20°C, the room temperature T_∞ is 293°K, and the kinematic viscosity of air is 0.15 cm^2/s, what are the values of U_c and δ from your scaling for a window of height H = 1m?

g. If our window is square (e.g., W = 1m too), and the thermal conductivity of air k is 0.026 W/m°K, what is the scaling for the energy loss in watts? Note that this is just the scaling, of course – it turns out that if you solve the whole problem (including the energy equation, needed to get the temperature profile) your "unknown O(1) constant" is about 0.5 for air (and is a weak function of the Prandtl number).

Problem 2. (20 points) Lubrication – a defined flow rate problem!: Cavitation is the phenomenon which occurs when the absolute pressure in a liquid reaches the vapor pressure, and the liquid boils. For most liquids this is pretty close to zero. While usually seen in inertial flows, it is actually most easily visualized in viscous lubrication. Consider the geometry depicted below:



Two plates of length L in contact at one edge are separated by a very small angle α. For small angles the gap between the plates is given by h = α x, where x is the distance from the vertex. The plates are pulled apart at some rate $d\alpha/dt$.

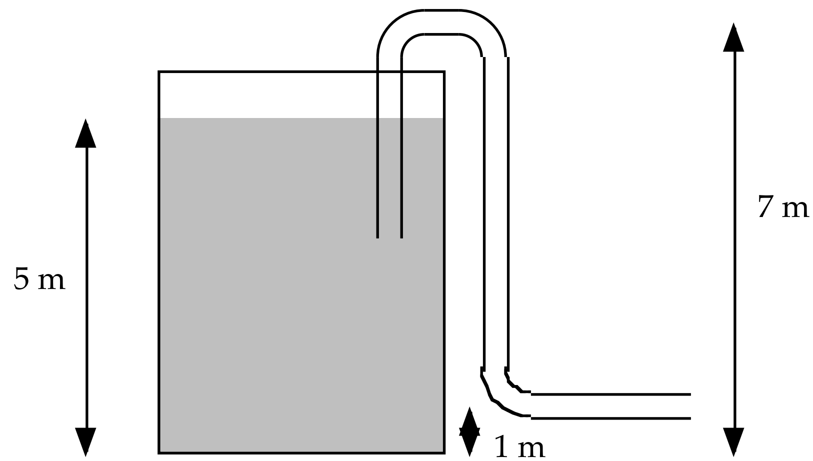
a. The fluid is incompressible (at least before cavitation occurs!) so the upward motion of the plate is balanced by fluid coming in from the x-direction. Using this, develop the expression for the x velocity averaged across the gap (y direction) as a function of $d\alpha/dt$, α, and x.

b. Write down the equation governing the velocity distribution and pressure gradient in the lubrication limit for some angular velocity $d\alpha/dt$ and scale it.

c. Evaluate the pressure distribution and develop an expression for the location x_c where the pressure falls to zero absolute (e.g., a gauge pressure of - p_{atm}) and the fluid cavitates.

d. Develop an integral expression for the force per unit width into the paper F/W (exerted at the outer edge) necessary to pry the plates apart. (Hint: Think of torque balances!) Scale it, but don't evaluate the integral to get the O(1) number…

Problem 3. (10 points) A Simple Siphon. Here we determine the flow rate of a siphon used to drain the tank depicted below. Water fills a tank to the height shown, and the siphon consists of 16 meters of 4cm ID pipe (it is filled with water too!).



a). Neglecting all frictional losses, what is the flow rate? Give your answer in liters/s.

b). Modify your answer by accounting for the head losses in the pipes and fittings. Correlations for friction factors in pipes and fittings are given below. You will probably need to do a couple of iterations to get the friction factor right. It helps to start the iterative calculation of the velocity with a reasonable guess for f_f since you know the other parameters. K values and head loss expressions are given below.

$$h_L = \frac{\langle u \rangle^2}{2g} \sum K + 4 \; f_f \frac{L}{D} \frac{\langle u \rangle^2}{2g}$$

$$f_f = \frac{16}{Re} \; ; \; \text{Re} < 2100 \qquad\qquad f_f \approx \frac{0.0791}{Re^{1/4}} \; ; \; 3000 < \text{Re} < 10^5$$

$$\frac{1}{\sqrt{f_f}} = 4.0 \log_{10}\left(\text{Re}\sqrt{f_f}\right) - 0.40 \; ; \; \text{Re} > 3000$$

Fitting	K value
sudden contraction	0.45
sudden expansion	1.0
90° elbow	0.7

Problem 4. (10 points) Pump Curves:

a. It is desired to pump 30 liters/sec of water from a pond to an elevation of 15 meters. How much head loss can we tolerate before the pump CP80i is no longer recommended for the job?

b. What is the RPM required to do the job?

c. What is the useful mechanical work done <u>by</u> the pump <u>on</u> the fluid?

d. What is the efficiency of the pump at the operating conditions?

e. How much head loss can we tolerate in the piping leading to the pump before it cavitates?

f. If we are pumping hot water at 60^0C (vapor pressure of 150mmHg vs. that at 25^0C of 23.7mmHg), how does your answer to part e change? (OK, a hint: mercury has a density of 13.56 times that of water…)