CBE 30355 Transport Phenomena I
Final Exam

December 11, 2023

Closed Books and Notes

Problem 1. (20 points) Thermal boundary layers. Consider the system depicted below. The fluid velocity is just **unidirectional simple shear flow in the x-direction**. The fluid enters with a temperature of zero (e.g., we've already subtracted off some reference temperature), and gets heated by a wall heat flux that is **linearly increasing** as we travel down the plate, e.g., $\mathbf{q_w = \lambda\, x}$ where λ is some constant. The thermal energy equation is given by:

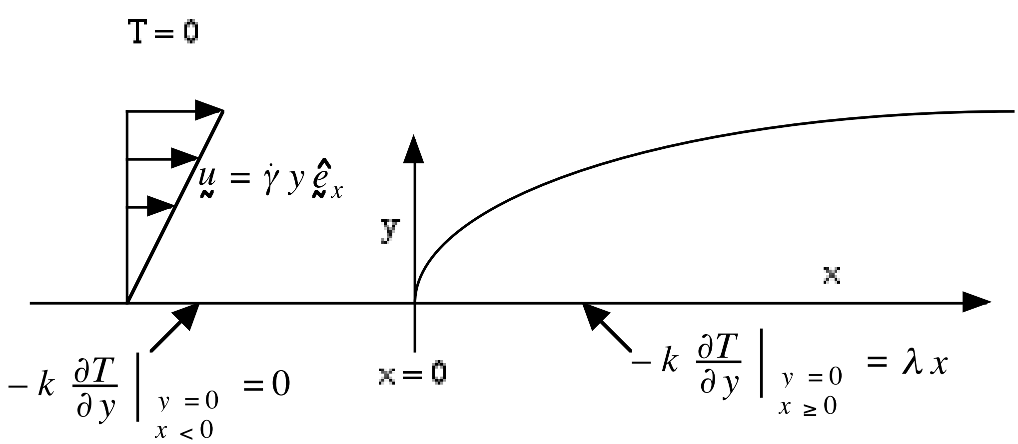
$$\frac{\partial T}{\partial t} + \underset{\sim}{u} \cdot \underset{\sim}{\nabla}\, T = \alpha\, \nabla^2\, T$$

with boundary conditions:

$$-k \left.\frac{\partial T}{\partial y}\right|_{\substack{y=0 \\ x \geq 0}} = \lambda\, x \quad , \quad -k \left.\frac{\partial T}{\partial y}\right|_{\substack{y=0 \\ x \leq 0}} = 0 \quad , \quad T\big|_{y \to \infty} = 0$$

a). Render the governing equation and boundary conditions dimensionless using a length scale L in the x-direction, and determine the conditions under which we can expect a thin boundary layer in the y-direction (e.g., how big does L have to be?). You may assume steady-state, with no variation in the z-direction.

b). Show that the thermal boundary layer equations yield a self-similar solution, obtaining the similarity rule and similarity variable in canonical form. Using this, determine the temperature at the plate as a function of x to within some undetermined multiplicative constant. Note that you don't have to get the transformed ODE or solve it to do this!



Problem 2. (20 points) Dimensional Analysis/Stokes flow. At the start of the pandemic I was asked by the Provost to evaluate whether a face shield (favored by the law school faculty) would serve as an adequate substitute for a face mask in protection against the emission or inhalation of droplets. Unfortunately, the answer was no. Because a shield is solid, the only way it can filter out droplets emitted during speech is via inertial impaction: because of their mass they do not exactly follow the streamlines of the air deflected by the shield, and instead (if they are big enough) impact on the surface. Here we examine this phenomenon by looking at a simpler one-dimensional analog.

Consider a spherical droplet of radius a which initially moves with the velocity U_0 of the fluid (the exhaled air). At time $t = 0$ you stop the flow (corresponding to deflection by the face shield in the real problem), but due to its mass the droplet keeps going for a while. Inertial impaction occurs if the final displacement relative to the fluid is greater than the distance to the face shield.

a. The displacement of the droplet Δx depends on the density of the drop ρ, the density of the fluid (air) ρ_a, the viscosity of the air μ, the radius of the drop a, and the initial velocity U_0. Using dimensional analysis, determine the dimensionless groups the displacement depends on.

b. The result in part a isn't terribly useful, as there are too many groups! If the droplets are really small, however, (and the ones we are most worried about are really really small!) their motion is governed by Stokes flow (low Re). In this case, the density of the air is negligible and the displacement Δx is proportional to U_0 (e.g., due to the linearity of the governing flow equations at low Re). Use this to strengthen your dimensional analysis down to a single group.

c. This still isn't good enough, as we need to actually solve the problem to get the "O(1) constant". Using Newton's Second Law (e.g., force = mass * acceleration!) and Stokes Law (hint: remember 6π?) for the drag on a sphere, set up and solve for the time-dependent velocity and displacement of a droplet.

d. If the initial velocity is 100cm/s, the viscosity of air is 1.181×10^{-4} g/cm s, and the distance to the face shield is 5 cm, quantitatively estimate the diameter of the droplets which could be captured by inertial impaction. The problem is that the droplets emitted by speech are smaller than this, and hence they would get away…

Problem 3. (20 points) Unidirectional Flows: In a common blood screening technique for cholesterol and glucose levels a technician pokes a hole in your finger to get a drop of blood. A bit of this blood is drawn into a capillary which is then inserted into an analysis machine. The blood is drawn into the capillary by surface tension as depicted below. If the surface tension of blood is $\Gamma = 56$ dynes/cm, the viscosity of blood is 4cp, the radius of the capillary is $a = 50\mu$m, and the length $L = 5$cm, how long does it take for the tube to fill? Recall that the capillary pressure for a fully wetted *circular* tube of radius a is $\Delta p_{cap} = 2\Gamma/a$.

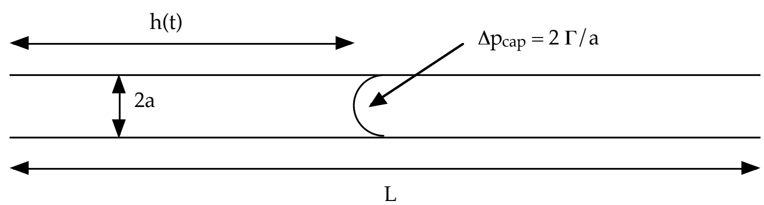
(Hint: Inertial effects are negligible for this problem, and the pressure differential is applied only over the filled length h of the capillary, which changes in time at a rate equal to the average axial velocity!)

a. Write down the governing equations and boundary conditions, as well as the equation governing the time dependent filled length h.

b. Render the equations dimensionless to determine how the filling time depends on the parameters of the problem.

c. Solve the problem to determine when the capillary is completely filled in terms of the parameters of the problem.

d. Plug in the numbers to get the final numerical value.



$$\rho\left(\frac{\partial u_z}{\partial t}+u_r\frac{\partial u_z}{\partial r}+\frac{u_\theta}{r}\frac{\partial u_z}{\partial \theta}+u_z\frac{\partial u_z}{\partial z}\right)=-\frac{\partial p}{\partial z}+\mu\left[\frac{1}{r}\frac{\partial}{\partial r}\left(r\frac{\partial u_z}{\partial r}\right)+\frac{1}{r^2}\frac{\partial^2 u_z}{\partial \theta^2}+\frac{\partial^2 u_z}{\partial z^2}\right]+\rho g_z$$

Problem 4. (20 points) Pumped Hydroelectric Energy Storage (PHES) is the most widely used energy storage method in the world. The idea is that when your electricity cost is low (e.g., when the sun is shining on all your solar cells or the wind is blowing on your wind turbines) you use the excess generating capacity to pump water to a high elevation, and when it is more costly (or when the sun isn't shining) you run the system backwards to get your electricity back. These systems can be huge: currently the largest in the world is in Bath, Virginia with a generating capacity of 3GW, enough to power 750,000 homes. Large scale systems are quite cost efficient, with a round trip energy efficiency of 70% to 80% and a relatively low capital cost/GW.

While large systems are economically viable (provided you happen to have a nice mountain with a water source nearby), they have also been proposed (and actually implemented) for individual buildings – but they are not nearly as cost effective. Here we examine some of the numbers associated with a PHES installation in an apartment building in France where one was installed (the Goudemand Residence Complex).

a. The two reservoirs (an open reservoir on the roof and storage tanks in the basement) are separated by 30m in elevation and have an exchange volume of 50m^3. Based on this, what is the total amount of electrical energy that the system can provide (assume that the turbine system converting the potential energy of the water back to electricity is about 75% efficient)? Give your answer in kWhr (unit conversion 1kWhr = 3.6x10^6 J).

b. The pumping system is required to pump water from the basement to the roof at a flow rate of 4 liters/s. How many hours would it take to "fully charge" this battery?

c. The pipe system has not been disclosed, however we shall take it to be 50 meters of 2 inch schedule 40 PVC pipe (ID 5.25 cm), plus a safety grate, a sudden expansion (e.g., the "1"), 8 90° elbows, two T's straight through and an open gate valve. Based on this, what is the total head that the pump has to supply?

d. Mark the operating point on the pump curve below (you will have to put your name on the exam paper and turn it in with the bluebook) and determine the required energy input to the pump. What fraction of this energy actually gets stored as potential energy? What is the return trip efficiency of the system (e.g., electricity out / electricity in, assuming the generation side is 75% efficient)?

e. What is the maximum head loss leading to the pump which could be tolerated before violating NPSHR and cavitating the pump?

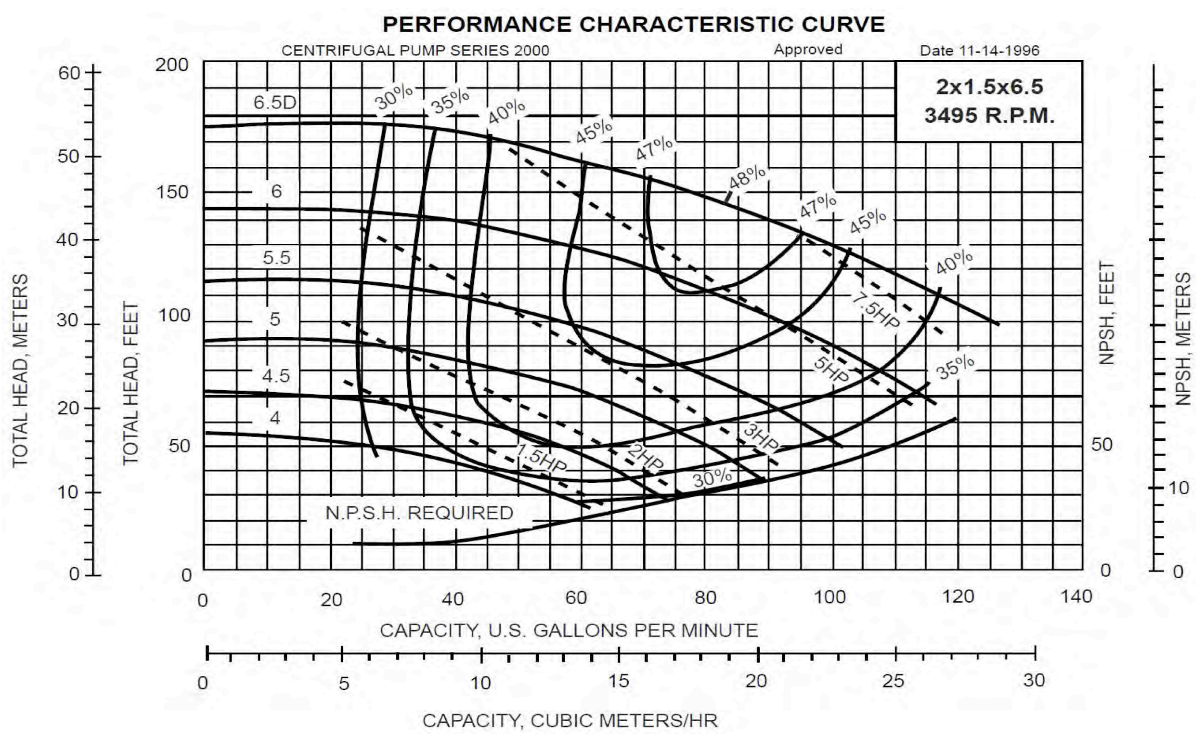
f. This apartment building was really a social experiment in energy utilization rather than trying to be cost effective, however we can look at the economics. To calculate the economics we shall assume as a best case scenario that the pump energy is "free" (it is excess energy from wind turbines and solar cells in this installation) and that the reservoir is fully cycled through the system on average once per day. If the cost of electricity is \$0.15/kWhr, what is yearly value of the energy stored by the system? The Goudemand PHES system was estimated to cost \$40k. How many years would it take to recover the capital cost under this idealized (no recurring maintenance or energy costs) scenario? (Hint: get it right, but don't be disturbed if you find it is a really long time…)

$$h_L = \frac{\langle u \rangle^2}{2g} \sum K + 4 \; f_f \frac{L}{D} \frac{\langle u \rangle^2}{2g}$$

$$f_f = \frac{16}{Re} \;;\; \mathrm{Re} < 2100 \qquad\qquad f_f \approx \frac{0.0791}{Re^{1/4}} \;;\; 3000 < \mathrm{Re} < 10^5$$

$$\frac{1}{\sqrt{f_f}} = 4.0 \log_{10}\left(\mathrm{Re}\sqrt{f_f}\right) - 0.40 \;;\; \mathrm{Re} > 3000$$

Fitting	K value
Safety grate	1.5
sudden expansion	1.0
90° elbow	0.7
T (straight thru)	0.4
Gate valve (open)	0.15



Note: This pump curve gives the power requirement in HP (unit conversion: 1 HP = 0.7457 kW). The curved lines are pump efficiencies – very convenient. The flow rate is in gal/min (unit conversion: 1 liter/s = 15.85 gal/min). And, of course, 1 meter = 3.28 ft.