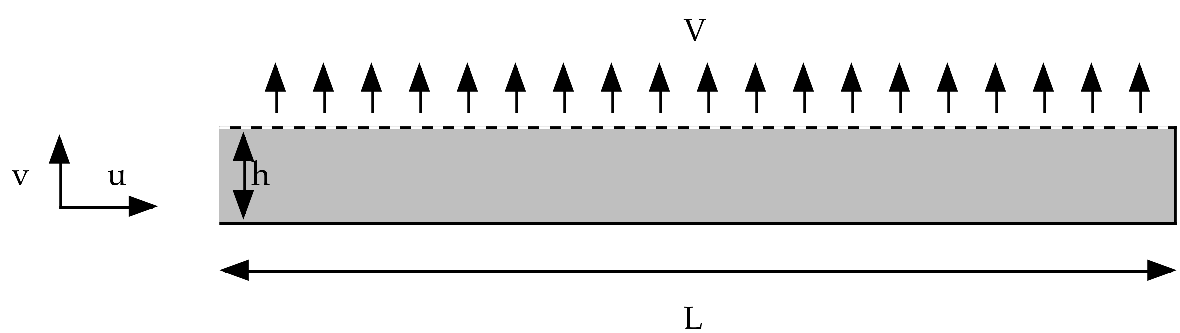
CBE 30355 Transport Phenomena I
Final Exam

December 13, 2016

Closed Books and Notes

Problem 1. (20 points) Quasi-parallel flows. Earlier this semester I was asked to evaluate a microfluidic plasma skimming device that was under consideration for licensing. While the complete system was more complicated, at its core the separator consisted of a channel where the top wall was porous as depicted below. Whole blood is drawn into the channel from the left end by the removal of plasma through the porous plate at the top. To determine the fate of the red blood cells, it is necessary to know the complete velocity profile and stream function (the cells tend to settle away from the upper surface, allowing cell free plasma to be drawn off – but you have to know the velocity profile of the fluid to figure out if they settle fast enough). Here we determine this velocity profile! (Note: while this problem is *similar* to the problem on the second hour exam, it is ***not identical!!!***).



a. By making the quasi-parallel flow approximation (and using a mass balance!) determine the equations and boundary conditions governing the velocities in the x and y directions. Render these dimensionless.

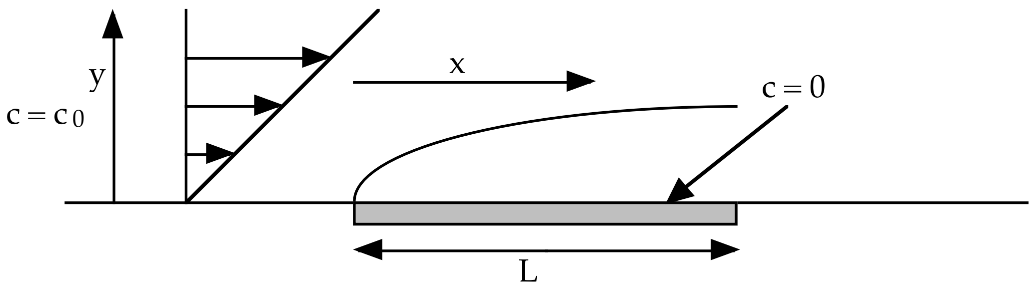
b. Solve these equations to determine the dimensionless velocity along the channel. If you remember the relation between centerline and average velocities in this geometry you can use it…

c. Solve for the dimensionless velocity in the y direction.

d. Determine the streamfunction ψ, taking the value of ψ to be zero at the bottom of the channel.

Contours of constant ψ could be plotted to determine the trajectories that the fluid elements follow, and the trajectories of the settling red blood cells (integrating $dx/dt = u$ and $dy/dt = v - U_s$) would be used to determine their fate in the system, but don't do this here!

Problem 2. (20 points) Scaling analysis of boundary layer flows. A popular method for measuring instantaneous wall shear stresses in turbulent flows is the use of an electrochemical probe. The fluid is doped with some reactant at concentration c_0 which decomposes electrochemically at an electrode flush with the wall. The reaction is essentially instantaneous, so the concentration of reactant at the electrode is zero, and the reaction rate is determined by the rate with which the stuff diffuses to the wall. By measuring the current, we can determine the reaction rate and hence the integrated mass flux to the electrode. The mass flux gets related to the shear stress because diffusive transport of mass is so slow relative to momentum transport that the mass transfer boundary layer essentially samples the linear shear flow right at the wall. We thus have the following problem:



$$\underset{\sim}{u} \cdot \underset{\sim}{\nabla} c = D \nabla^2 c \quad ; \quad \underset{\sim}{u} \sim \underset{\sim}{\hat{e}}_x \frac{\tau_w}{\mu} y$$

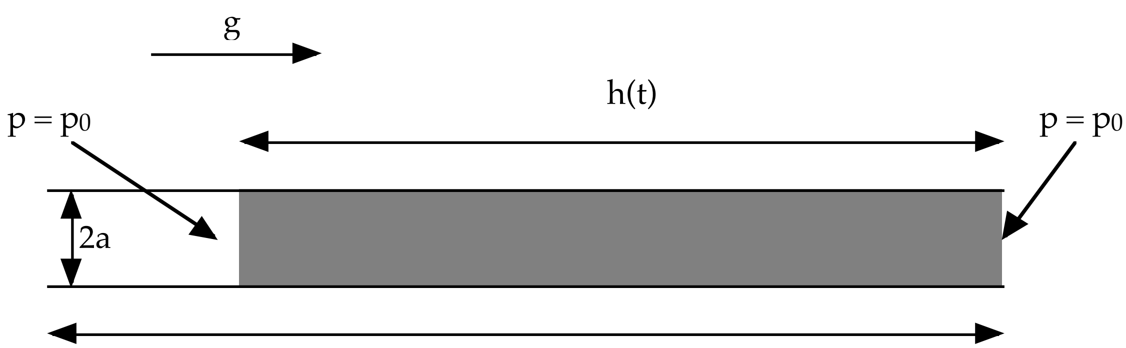
$$N\Big|_{y=0} = -D \frac{\partial c}{\partial y}\Big|_{y=0} \quad ; \quad I = W \int_0^L N\Big|_{y=0} dx$$

a. Using scaling analysis, determine how the current per unit width I/W scales with the parameters of the problem (e.g., τ_w, D (the diffusion coefficient), μ, c_0, and L).

b. Estimate the time resolution of such a probe (e.g., how long would it take for the signal to approach steady state in the presence of shear transients). Choose L = 1mm, D = 10^{-6} cm^2/s, τ_w = 100 $dynes/cm^2$, and the properties of water.

c. The current is only related to the shear stress if the boundary layer approximation holds. In general, we would like to make the probe as short as possible to get the best time resolution possible. What is the shortest length the probe can be?

Problem 3. (20 pts) Scaling/Unidirectional flows: Consider a **vertical** straw of length L and radius a containing a liquid with viscosity μ and density ρ as depicted below (the diagram **is sideways so it fits on the page**!!!). The liquid is initially at rest (we put our finger over the top of the straw). At time t=0 we remove our finger and allow the liquid to drain out of the straw due to gravity. The length of the straw filled with fluid at any time t is given by h. In this problem we wish to determine the time T_d required to empty the straw - e.g., how long does it take for h to reach zero.



a. A friend argues that T_d is a function of L, a, g, and fluid properties. Use dimensional analysis to determine a dimensionless expression for T_d in terms of these parameters (e.g., construct relevant π groups).

b. Using the unidirectional flow approximation, write down the differential equation governing the fluid velocity (hint: unsteady, in general), keeping only the non-zero terms. Also write down the relation between the velocity and the change in length with time dh/dt of the column of fluid in the straw of length L. Write down all relevant boundary conditions and initial conditions.

b. Scale the equations for HIGH Reynolds numbers, and determine the (unknown) characteristic drainage time t_c in this limit. What boundary/initial condition(s) have to be thrown out in this limit, and what dimensionless group of parameters has to be small?

c. Solve for the drainage time T_d in this limit.

d. Scale the equations for LOW Reynolds numbers, and determine the characteristic drainage time t_c in this limit.

The following equations *may* be helpful:

$$\frac{1}{r}\frac{\partial(r\,u_r)}{\partial r}+\frac{1}{r}\frac{\partial u_\theta}{\partial \theta}+\frac{\partial u_z}{\partial z}=0$$

$$\rho\left(\frac{\partial u_z}{\partial t}+u_r\frac{\partial u_z}{\partial r}+\frac{u_\theta}{r}\frac{\partial u_z}{\partial \theta}+u_z\frac{\partial u_z}{\partial z}\right)=-\frac{\partial p}{\partial z}+\mu\left[\frac{1}{r}\frac{\partial}{\partial r}\left(r\frac{\partial u_z}{\partial r}\right)+\frac{1}{r^2}\frac{\partial^2 u_z}{\partial \theta^2}+\frac{\partial^2 u_z}{\partial z^2}\right]+\rho g_z$$

Problem 4. (20 points) It is desired to pump water from a pond into a storage tank using the piping network below (pipe lengths not to scale). The requirements are a flow of 20 liters/s, and the proposed pipe diameter is 7.8 cm ID (e.g., a 3" schedule 40 PVC pipe). The total pipe length is 50 meters.

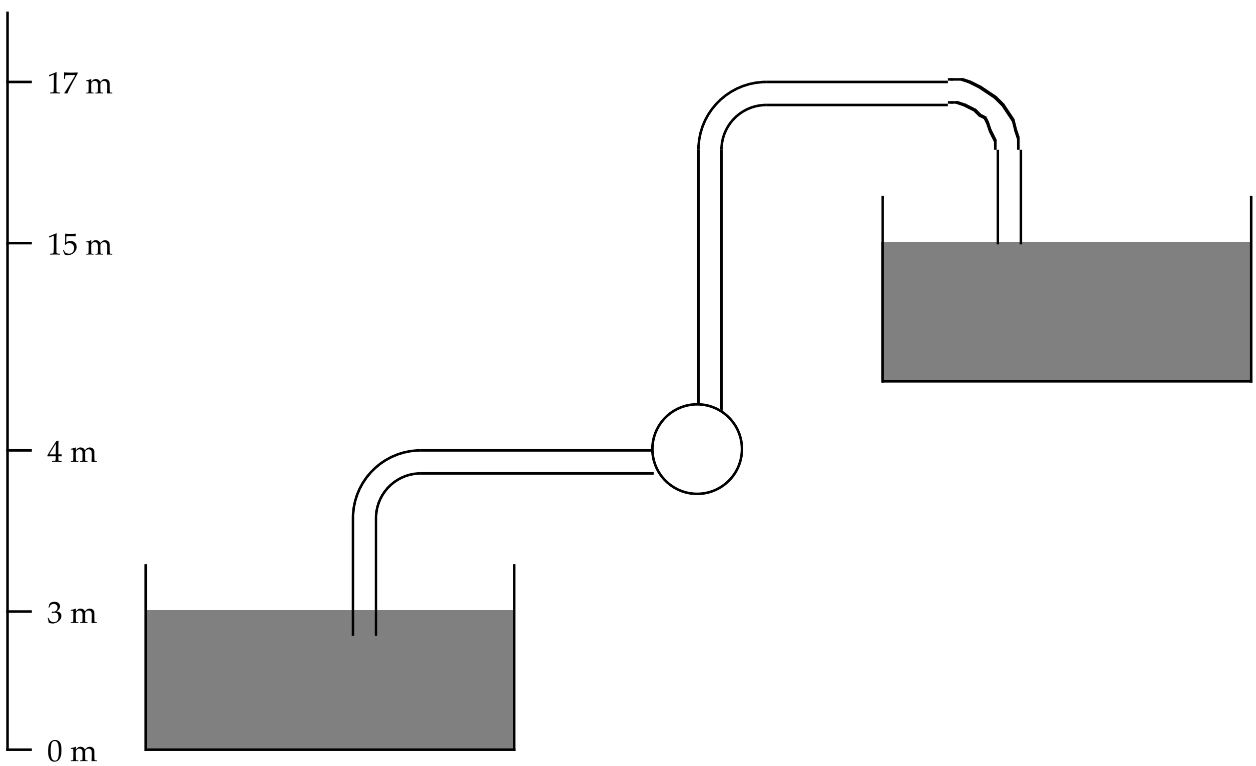
a. Calculate the total head required of the pump. Correlations for friction factors in pipes and fittings are given on the next page.

b. It is proposed to use the pump CP-80i (pump curve is on the next page). Identify the operating point on the curve, and use this to determine the required RPM and power requirement.

c. What is the efficiency of the pump under these conditions?

d. How close to the surface of the pond (vertical elevation) is it necessary locate the pump? (you can ignore f_f for this part because the pipe is short, but include other losses!)

e. It is suggested that operating costs could be significantly reduced if a larger pipe diameter is employed. If the cost of electricity is 0.10 $/kwhr (current Indiana rates), what would be the maximum monthly savings that going to a (much) larger pipe could possibly result in? Note that you wouldn't actually have to make it all that much bigger – the losses go roughly as D^{-4}!



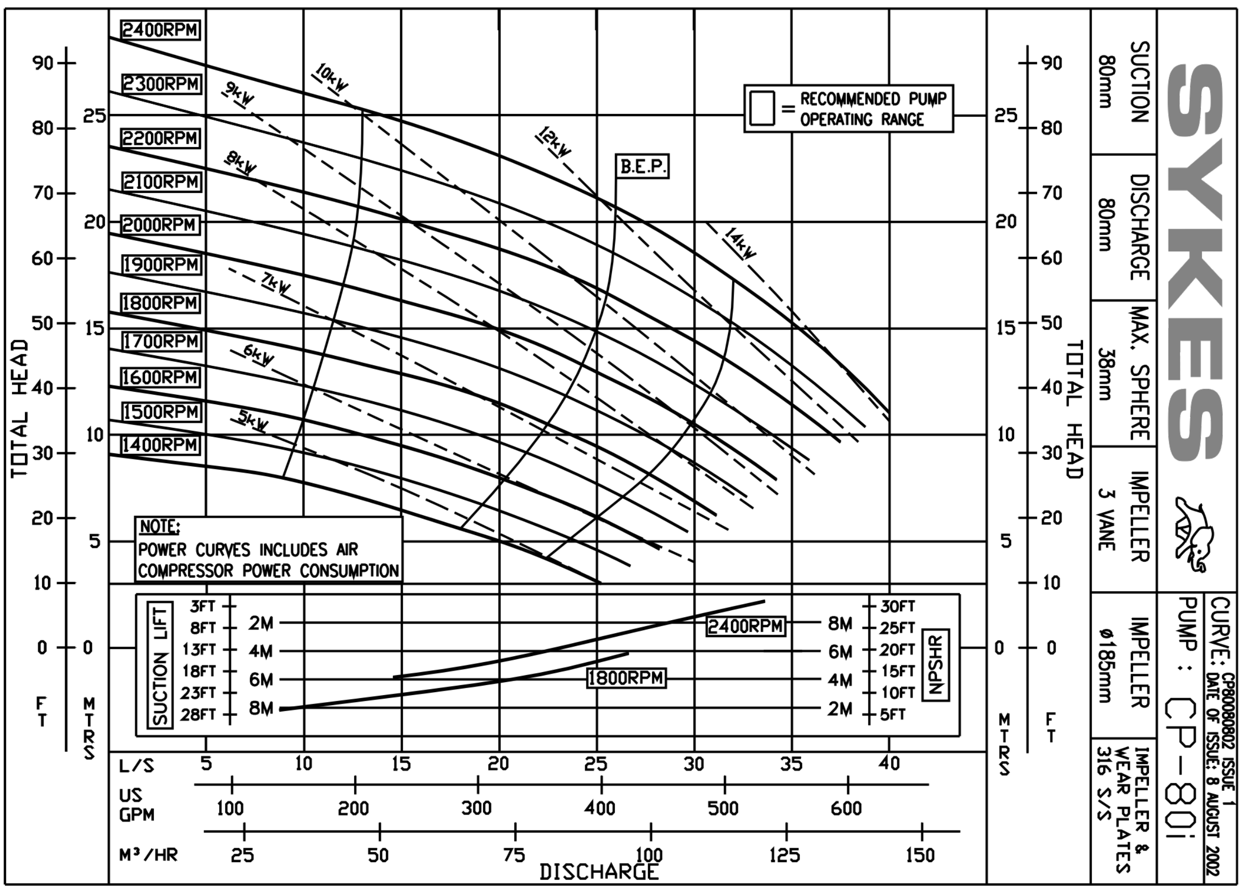
Note: Drawing not to scale (except elevations)!

$$h_L = \frac{\langle u \rangle^2}{2g} \sum K + 4 f_f \frac{L}{D} \frac{\langle u \rangle^2}{2g}$$

$$f_f = \frac{16}{Re} \;;\; \mathrm{Re} < 2100 \qquad\qquad f_f \approx \frac{0.0791}{Re^{1/4}} \;;\; 3000 < \mathrm{Re} < 10^5$$

$$\frac{1}{\sqrt{f_f}} = 4.0 \log_{10}\left(\mathrm{Re}\sqrt{f_f}\right) - 0.40 \;;\; \mathrm{Re} > 3000$$

Fitting	K value
sudden contraction	0.45
sudden expansion	1.0
90° elbow	0.7



Problem 5). (20 points) Short Answer:

1. What is stall and why is it bad (be specific…)?

2. What were the three ways of remediating it discussed in class?

3. Give a physical description of the Reynolds stress (e.g., where does it come from, and how is it defined?).

4. For a shear stress of 16 dynes/cm^2 in the turbulent flow of water through a pipe, about how rough does the pipe wall have to be before it influences the flow?

5. What do the dimples on a golf ball do?

6. What is the Von Karman Momentum Balance useful for, and why?

7. What is the difference between an orifice meter and a venturi meter?

8. What is electroosmosis, and where does it come from?

9. What is the "Law of the Wall"? What is the key approximation that leads to it?

10. What is Stokes Paradox? Why does it affect flow in a shower drain?