

AI Project 4: AI Use in Finals Prep

Due: 11/18/25

Assignment

In this project you are going to use AI to help prepare for the final Transport exam. Four problems are given below that are taken off old final exams. For each of these problems:

1. **Restatement of Problem:** Have AI provide a concise restatement of the problem being solved.
2. **AI Solution in $\text{\LaTeX}$:** Use AI to prepare a $\text{\LaTeX}$ -formatted solution to the problem (feel free to solve it yourself first!). Edit the AI output as needed for clarity, correctness, and formatting.
3. **Study Guide:** Provide a short study guide for the concepts behind the problem. This should include:
 - 2–3 key concepts students must master to solve the problem.
 - Identify which class lecture is most applicable to the problem
 - At least one additional practice problem or variation (state the problem, do not solve it).
4. **Screen Recording.** Create a narrated screen recording (similar to recorded lectures). In your recording:
 - Explain the solution to each problem.
 - Highlight the conceptual analysis and study guide elements.
 - Emphasize your reasoning, not just reading the AI-generated text.

Upload the recording to Panopto and insert the link(s) into both your $\text{\LaTeX}$ source and PDF.

Deliverables

- A compiled PDF report containing:
 - a. A restatement of each problem.
 - b. Solutions in $\text{\LaTeX}$ for each problem.
 - c. Concept analysis and study guide for each problem.
 - d. Panopto link(s) to your screen recording.
- Your Panopto recording uploaded and linked into the PDF.

Guidelines

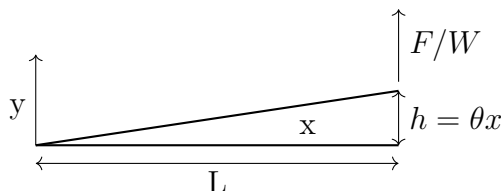
- Keep your total recording time under 40 minutes. Aim for about 5–7 minutes per problem.
- It is useful (and faster!) to do these problems one at a time with separate files and recordings for each. The links to the recordings need to be added into the $\text{\LaTeX}$ after the recordings are generated. The individual $\text{\LaTeX}$ files should be combined (e.g., cut and paste with a single preamble and end of document command) to yield a single PDF at the end.
- Remember: the final will be closed-book and closed-AI. The purpose of this project is to use AI as a learning partner now, so you are prepared to work independently on the exam.
- Professionalism in both your $\text{\LaTeX}$ write-up and recorded presentation will be part of the evaluation.

The Problems

Problem 1

Lubrication and Cavitation in Separating Plates

Cavitation is the phenomenon which occurs when the absolute pressure in a liquid reaches the vapor pressure, and the liquid boils. For most liquids this is pretty close to zero. While usually seen in inertial flows, it is actually most easily visualized in viscous lubrication. Consider the geometry depicted below:



Two plates of length L in contact at one edge are separated by a very small angle θ . For small angles the gap between the plates is given by

$$h(x) = \theta x$$

where x is the distance from the vertex. The plates are pulled apart at some rate $\frac{d\theta}{dt}$.

- Using lubrication analysis, determine the location x_c where the pressure falls to zero absolute (e.g., a gauge pressure of $-p_{\text{atm}}$) and the fluid cavitates.
- What is the force per unit width into the paper F/W (exerted at the outer edge) necessary to pry the plates apart for some angular velocity $\frac{d\theta}{dt}$? (Hint: Think of torque balances! You may leave this in terms of an integral if you wish.)

Problem 2

Inertial Impaction and Face Shields

Last summer I was asked to evaluate whether a face shield would serve as an adequate substitute for a face mask in protection against the emission or inhalation of droplets. Unfortunately, the answer was no. Because a shield is solid, the only way it can filter out droplets emitted during speech is via inertial impaction: because of their mass they do not exactly follow the streamlines of the air deflected by the shield, and instead (if they are big enough) impact on the surface. Here we examine this phenomenon by looking at a one-dimensional analog.

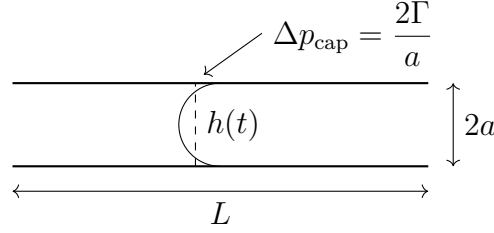
Consider a spherical droplet of radius a which initially moves with the velocity U_0 of the fluid (the exhaled air). At time $t = 0$ you stop the flow (corresponding to deflection by the face shield in the real problem), but due to its mass the droplet keeps going for a while. Inertial impaction occurs if the final displacement relative to the fluid is greater than the distance to the face shield.

- a. The displacement of the droplet x depends on the density of the drop ρ , the density of the fluid (air) ρ_a , the viscosity of the air μ , the radius of the drop a , and the initial velocity U_0 . Using dimensional analysis, determine the dimensionless groups the displacement depends on.
- b. The result in part (a) isn't terribly useful, as there are too many groups! If the droplets are really small, however (and the ones we are most worried about are really really small!), their motion is governed by Stokes flow (low Re). In this case, the density of the air is negligible and the displacement x is proportional to U_0 (e.g., due to the linearity of the governing flow equations). Use this to strengthen your dimensional analysis down to a single group.
- c. This still isn't good enough, as we need to actually solve the problem to get the " $O(1)$ " constant. Using Newton's Second Law (e.g., force = mass $\times$ acceleration) and Stokes' Law (hint: remember $6\pi \dots$) for the drag on a sphere, set up and solve for the time-dependent velocity and displacement of a droplet.
- d. If the initial velocity is 100 cm/s, the viscosity of air is 1.81×10^{-4} g/cm $\cdot$ s, and the distance to the face shield is 5 cm, estimate the diameter of the droplets which could be captured by inertial impaction. The problem is that the droplets emitted in speech are smaller than this, and thus they get away...

Problem 3

Capillary Filling of a Tube by Surface Tension

In the annual University sponsored health screenings for faculty and staff, technicians run a blood test to measure cholesterol and glucose levels. What they do is poke a hole in your finger to get a drop of blood and then draw the blood into a capillary which is then inserted into the analysis machine. The blood is drawn into the capillary by surface tension as depicted below. If the surface tension of blood is $\Gamma = 56$ dynes/cm, the viscosity of blood is 4 cP, the radius of the capillary is $a = 50$ μ m, and the length $L = 5$ cm, how long does it take for the tube to fill?



(Hint: Inertial effects are negligible for this problem, and the pressure differential is applied only over the filled length h of the capillary, which changes in time at a rate equal to the average axial velocity!)

- Write down the governing equations and boundary conditions, as well as the equation governing the time dependent filled length h .
- Render the equations dimensionless to determine how the filling time depends on the parameters of the problem.
- Solve the problem to determine when the capillary is completely filled in terms of the parameters of the problem.
- Plug in the numbers to get the final numerical value.

$$\rho \left(\frac{\partial v_z}{\partial t} + v_r \frac{\partial v_z}{\partial r} + \frac{v_\theta}{r} \frac{\partial v_z}{\partial \theta} + v_z \frac{\partial v_z}{\partial z} \right) = -\frac{\partial p}{\partial z} + \mu \left[\frac{1}{r} \frac{\partial}{\partial r} \left(r \frac{\partial v_z}{\partial r} \right) + \frac{1}{r^2} \frac{\partial^2 v_z}{\partial \theta^2} + \frac{\partial^2 v_z}{\partial z^2} \right] + \rho g_z$$

$$\frac{\partial \rho}{\partial t} + \frac{1}{r} \frac{\partial}{\partial r} (r \rho v_r) + \frac{1}{r} \frac{\partial}{\partial \theta} (\rho v_\theta) + \frac{\partial}{\partial z} (\rho v_z) = 0$$

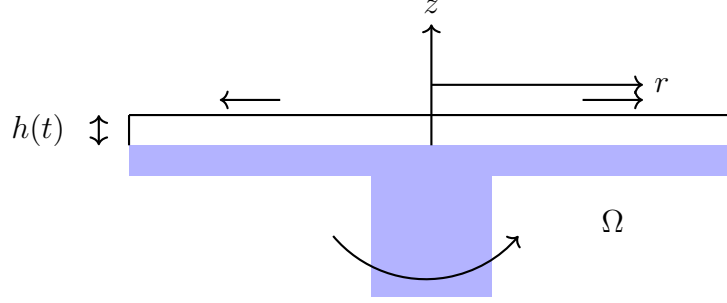
Problem 4

Spin Coating and Thin Film Dynamics

Spin coating is a common way to produce a thin film on a surface: a disk of radius R is coated with a viscous liquid of kinematic viscosity ν , and rotated with angular velocity Ω . Centrifugal forces thin the fluid layer over time, leaving a final thickness $h_f \ll R$.

- For thin fluid layers the rotational velocity is just $u_\theta = \Omega r$ over the entire film (i.e., it moves with the rotational velocity of the disk). Using this, determine the differential equations and boundary conditions which govern the radial and vertical velocity distribution in the film, and the equation for the change in film thickness over time. (Hint: How does the change in thickness with time relate to u_z at $z = h(t)$?) It is appropriate to use h_f , the final thickness, as the vertical length scale.

- b. Using scaling analysis, determine how long the spin coating process takes as a function of the parameters of the problem to within the usual unknown $\mathcal{O}(1)$ constant.
- c. Explicitly solve for the velocity distribution and the height as a function of time to get the $\mathcal{O}(1)$ constant. You may take the initial height to be H , where $H/h_f \gg 1$.



$$\frac{1}{r} \frac{\partial}{\partial r} (r u_r) + \frac{1}{r} \frac{\partial u_\theta}{\partial \theta} + \frac{\partial u_z}{\partial z} = 0$$

$$\rho \left(\frac{\partial u_r}{\partial t} + u_r \frac{\partial u_r}{\partial r} + \frac{u_\theta}{r} \frac{\partial u_r}{\partial \theta} - \frac{u_\theta^2}{r} + u_z \frac{\partial u_r}{\partial z} \right) = -\frac{\partial p}{\partial r} + \mu \left[\frac{\partial}{\partial r} \left(\frac{1}{r} \frac{\partial (r u_r)}{\partial r} \right) + \frac{1}{r^2} \frac{\partial^2 u_r}{\partial \theta^2} - \frac{2}{r^2} \frac{\partial u_\theta}{\partial \theta} + \frac{\partial^2 u_r}{\partial z^2} \right] + \rho g_r$$

Rubric

Your grade will be based on the following criteria:

1. **Accuracy of Solutions (35%)** Correctness of the $\text{\LaTeX}$ -formatted solutions and appropriateness of any edits to AI output.
2. **Depth of Conceptual Analysis (20%)** Clarity in identifying the key transport phenomena concepts each problem is testing.
3. **Usefulness of Study Guide (15%)** Clarity of explanations, and inclusion of at least one meaningful practice problem per question.
4. **Clarity and Professionalism of $\text{\LaTeX}$ Write-Up (10%)** Organization, readability, formatting, and integration of links.
5. **Quality of Recorded Presentation (20%)** Effectiveness of narration and clarity of reasoning.