CBE 30235: Introduction to Nuclear Engineering Problem Set 2: Decay Kinetics and Cosmogenic Sources

Due: Friday, January 30, 2025 (via GradeScope at Midnight)

Problem 1: The Kinematics of Carbon-14 Production

In class, we discussed the primary production channel for cosmogenic ^{14}C in the upper atmosphere:

$$^{14}_{7}\mathrm{N} + {}^{1}_{0}n \longrightarrow {}^{14}_{6}\mathrm{C} + {}^{1}_{1}H$$

(a) Calculate the Q-value of this reaction in MeV. Use the following neutral atomic masses: $M(^{14}\mathrm{N}) = 14.003074$ u, $m_n = 1.008665$ u, $M(^{14}\mathrm{C}) = 14.003242$ u, and $M(^{1}\mathrm{H}) = 1.007825$ u.

(b) Is this reaction exothermic (releasing energy) or endothermic (requiring threshold energy)?

(c) If a thermal neutron (with negligible kinetic energy, $E \approx 0.025$ eV) is captured by a stationary ^{14}N nucleus, what is the total kinetic energy shared by the resulting ^{14}C and the proton?

Problem 2: The Voyager 1 "Shielding" Observation

Data from the Voyager 1 spacecraft showed that the flux of Galactic Cosmic Rays (GCRs) increased by approximately a factor of 10 upon crossing the Heliopause and entering interstellar space.

- If the production rate of ^{14}C on Earth is currently $P \approx 2.0$ atoms/(cm$^2\cdot$ s) given the Sun's current magnetic shielding, estimate what the production rate would be if the Sun suddenly stopped producing a solar wind (assume P scales linearly with incident GCR flux).
- Explain in 1–2 sentences how a period of **high solar activity** (maximum sunspots and strong solar wind) affects the Carbon-14 "clock" on Earth. Does it lead to an over-production or under-production of ^{14}C relative to the average?

Problem 3: The "Moly Cow" (Transient Equilibrium)

In a hospital Technetium-99m generator, the parent ^{99}Mo (λ_P) decays into the daughter ^{99m}Tc (λ_D). The half-lives are $T_{1/2,P} = 66$ hours and $T_{1/2,D} = 6.0$ hours.

(a) Calculate the decay constants λ_P and λ_D in units of hr^{-1}.

(b) Since $T_{1/2,P} > T_{1/2,D}$ (but not effectively infinite), the system reaches **Transient Equilibrium** rather than Secular Equilibrium.

Show that the ratio of activities A_D/A_P at long times ($t \gg 1/\lambda_D$) approaches a constant value greater than 1. Calculate this ratio.

(c) Calculate the time t_{max} (in hours) at which the daughter activity ^{99m}Tc reaches its maximum value after the "cow" has been milked (i.e., assuming $A_D(0) = 0$).

Problem 4: Spallation vs. Fission

A 1.0 GeV proton strikes a ^{14}N nucleus in the upper atmosphere, triggering a spallation event.

- Contrast this process with the thermal fission of ^{235}U in a power reactor. Specifically, compare the **incident particle energy** required and the **types of secondary particles** produced.
- Why do we use high-Z materials like Mercury ($Z = 80$) or Tungsten ($Z = 74$) at spallation neutron sources (like the SNS at Oak Ridge) instead of low-Z materials like Nitrogen to maximize neutron yield?

Problem 5: Atmospheric Attenuation

The Earth's atmosphere has a total areal density (thickness) of $\Sigma_{atm} \approx 1030$ g/cm^2. The mean free path (λ_{mfp}) of a high-energy nucleon in air is approximately 90 g/cm^2.

- Calculate the probability that a primary GCR proton will reach sea level without undergoing a single nuclear collision. (Hint: Use $P(x) = e^{-x/\lambda}$).
- Based on your result, explain why the production of cosmogenic isotopes (like ^{14}C and ^{10}Be) is concentrated in the stratosphere/upper troposphere rather than at the Earth's surface.