

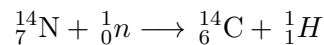
# CBE 30235: Introduction to Nuclear Engineering

## Problem Set 2: Decay Kinetics and Cosmogenic Sources

Due: Friday, January 30, 2025 (via GradeScope at Midnight)

### Problem 1: The Kinematics of Carbon-14 Production

In class, we discussed the primary production channel for cosmogenic  $^{14}\text{C}$  in the upper atmosphere:



- (a) Calculate the  $Q$ -value of this reaction in MeV. Use the following neutral atomic masses:  $M(^{14}\text{N}) = 14.003074$  u,  $m_n = 1.008665$  u,  $M(^{14}\text{C}) = 14.003242$  u, and  $M(^1\text{H}) = 1.007825$  u.
- (b) Is this reaction exothermic (releasing energy) or endothermic (requiring threshold energy)?
- (c) If a thermal neutron (with negligible kinetic energy,  $E \approx 0.025$  eV) is captured by a stationary  $^{14}\text{N}$  nucleus, what is the total kinetic energy shared by the resulting  $^{14}\text{C}$  and the proton?

### Problem 2: The Voyager 1 "Shielding" Observation

Data from the Voyager 1 spacecraft showed that the flux of Galactic Cosmic Rays (GCRs) increased by approximately a factor of 10 upon crossing the Heliopause and entering interstellar space.

- If the production rate of  $^{14}\text{C}$  on Earth is currently  $P \approx 2.0$  atoms/(cm<sup>2</sup> · s) given the Sun's current magnetic shielding, estimate what the production rate would be if the Sun suddenly stopped producing a solar wind (assume  $P$  scales linearly with incident GCR flux).
- Explain in 1–2 sentences how a period of **high solar activity** (maximum sunspots and strong solar wind) affects the Carbon-14 "clock" on Earth. Does it lead to an over-production or under-production of  $^{14}\text{C}$  relative to the average?

### Problem 3: The "Moly Cow" (Transient Equilibrium)

In a hospital Technetium-99m generator, the parent  $^{99}\text{Mo}$  ( $\lambda_P$ ) decays into the daughter  $^{99m}\text{Tc}$  ( $\lambda_D$ ). The half-lives are  $T_{1/2,P} = 66$  hours and  $T_{1/2,D} = 6.0$  hours.

- (a) Calculate the decay constants  $\lambda_P$  and  $\lambda_D$  in units of hr<sup>-1</sup>.
- (b) Since  $T_{1/2,P} > T_{1/2,D}$  (but not effectively infinite), the system reaches **Transient Equilibrium** rather than Secular Equilibrium.

Show that the ratio of activities  $A_D/A_P$  at long times ( $t \gg 1/\lambda_D$ ) approaches a constant value greater than 1. Calculate this ratio.

- (c) Calculate the time  $t_{max}$  (in hours) at which the daughter activity  $^{99m}\text{Tc}$  reaches its maximum value after the "cow" has been milked (i.e., assuming  $A_D(0) = 0$ ).

## Problem 4: Spallation vs. Fission

A 1.0 GeV proton strikes a  $^{14}\text{N}$  nucleus in the upper atmosphere, triggering a spallation event.

- Contrast this process with the thermal fission of  $^{235}\text{U}$  in a power reactor. Specifically, compare the **incident particle energy** required and the **types of secondary particles** produced.
- Why do we use high-Z materials like Mercury ( $Z = 80$ ) or Tungsten ( $Z = 74$ ) at spallation neutron sources (like the SNS at Oak Ridge) instead of low-Z materials like Nitrogen to maximize neutron yield?

## Problem 5: Atmospheric Attenuation

The Earth's atmosphere has a total areal density (thickness) of  $\Sigma_{atm} \approx 1030 \text{ g/cm}^2$ . The mean free path ( $\lambda_{mfp}$ ) of a high-energy nucleon in air is approximately  $90 \text{ g/cm}^2$ .

- Calculate the probability that a primary GCR proton will reach sea level without undergoing a single nuclear collision. (Hint: Use  $P(x) = e^{-x/\lambda}$ ).
- Based on your result, explain why the production of cosmogenic isotopes (like  $^{14}\text{C}$  and  $^{10}\text{Be}$ ) is concentrated in the stratosphere/upper troposphere rather than at the Earth's surface.