

# Design and analysis of a paraffin-chain Galileo-style thermometer

Prepared by ChatGPT for DTL

## Abstract

This report analyses a Galileo-style thermometer variant in which a small paraffin prism (square cross-section, rotated so one vertex points down) is tethered by a flexible steel chain to a wall anchor. The paraffin block's volumetric thermal expansion (much larger than water's) produces a change in buoyancy with temperature; that change lifts or releases a small portion of the chain, producing a measurable vertical motion. The task is to size the paraffin block so that the float moves **3 mm per °C**. Values for thermal expansion and densities are taken from EngineeringToolBox (see bibliography). Final numeric recommendations (volume, block dimensions, chain length, lead mass) are given together with sensitivity and uncertainty analysis. A TikZ figure and a materials table are included.

## 1 Short historical note

The original Galileo thermometer (late 16th/early 17th century, developed from early “thermoscopic” devices) employs sealed glass bulbs of differing densities that rise or fall as the surrounding liquid density changes with temperature; temperature readout is by observing which bulbs float or sink. Although commonly attributed to Galileo, the device was in fact developed later by members of the Accademia del Cimento in Florence, building on earlier concepts of thermoscopy [1].

The device presented here follows the same physical principle (buoyancy) but uses the large volumetric expansion of solid paraffin compared with the smaller expansion of water to produce a continuous displacement that can be read from a scale aligned with the block. This approach trades the discrete-bulb readout for a continuously moving float plus mechanical advantage derived from a tethered chain and a small ballast.

Related paraffin-based actuators, which exploit the relatively high thermal expansion and latent heat of paraffin, have been developed for engineering applications ranging from thermostats to solar tracking systems [2]. Thus, the present design lies at the intersection of historical thermometry and modern thermal actuator concepts.

## 2 Concept and assumptions

- The paraffin block is a small square prism (square cross-section, thickness given separately). In the figure the square is shown rotated 45 degrees so one vertex points downward; the chain and ballast attach at that bottom vertex.
- The chain is steel, with linear mass given by the user:  $m_{\text{air,chain}} = 0.083 \text{ g/cm}$  (that is 0.0083 kg/m in SI).
- The wall anchor is at approximately the same vertical level as the block center at the nominal room temperature (i.e., the chain hangs in a U between anchor and the block's bottom vertex).

- Small-angle / small-displacement linearization is appropriate: target sensitivity  $dz/dT = 3 \text{ mm}/^\circ\text{C}$  (i.e.  $0.003 \text{ m/K}$ ).
- Material data: for the numerical design I used EngineeringToolBox values (volumetric expansion for solid paraffin, water density and its thermal expansivity, and standard densities for steel and lead). Those sources are listed in the references.
- Geometry choices (anchor spacing, chain slack) are conservative estimates so the design fits in a vessel no taller than 20 cm (explicit assumptions are noted below).

### 3 Physical model and equations

Let

$V(T)$  be the paraffin block volume at temperature  $T$ ,

$\rho_w(T)$  be the water density at  $T$ ,

$\beta_p, \beta_w$  be the volumetric expansion coefficients of paraffin and water ( $1/\text{K}$ ).

The buoyant force (upward) on the block (plus anything rigidly attached to it whose displaced volume is included) is

$$B(T) = \rho_w(T) g V(T).$$

For small temperature changes (linear approximation)

$$\frac{dB}{dT} = g \left( \rho_w \frac{dV}{dT} + V \frac{d\rho_w}{dT} \right) = g \rho_w V (\beta_p - \beta_w),$$

because  $dV/dT = \beta_p V$  and  $d\rho_w/dT = -\rho_w \beta_w$ .

If the float rises by an amount  $dz$  it picks up (i.e. begins to support) an additional length  $dz$  of chain (model assumption explained below); the additional *force* that must be supplied (per unit vertical motion) is the (apparent) weight per unit length of chain in water:

$$\lambda = m'_{\text{chain}} g, \quad \text{where} \quad m'_{\text{chain}} = m_{\text{air,chain}} \left( 1 - \frac{\rho_w}{\rho_{\text{steel}}} \right)$$

is the chain's *effective* (apparent) mass per unit length when submerged (we treat the chain as thin steel so the displaced volume per unit length is  $m_{\text{air}}/\rho_{\text{steel}}$ ). Thus

$$\frac{d}{dT} (\text{downward force supplied to block}) = \lambda \frac{dz}{dT}.$$

Equilibrium of incremental changes gives

$$\frac{dB}{dT} = \lambda \frac{dz}{dT}.$$

Solving for the required paraffin volume  $V$  to obtain a specified  $\frac{dz}{dT}$ :

$$V = \frac{m'_{\text{chain}} \frac{dz}{dT}}{\rho_w (\beta_p - \beta_w)}.$$

This is the working formula used below.

## Model notes and justification

- The principal simplifying geometric assumption is that a vertical rise  $dz$  of the block corresponds to an additional supported chain length of approximately  $dz$ . For the typical geometry (anchor near block center height, short horizontal offset inside a small vessel), this is a conservative linear approximation: a local vertical displacement produces an equal (first-order) change in the chain length that must be supported by the float. If you want the exact chain geometry (catenary curve, horizontal offset), that can be added, but it changes results only at the tens-of-percent level for our small geometry and will not change the method.
- All expansions are taken at room temperature (20 °C) and linearized; the water expansion coefficient is temperature-dependent but for a design that reads near room temperature the linear approximation is acceptable.
- The chain linear mass in water used for the mechanical balance (mass per unit length in water) is different from the chain mass used in the overall static balance (total chain mass transmitted to the float). The equilibrium (nominal centering) uses true masses (air masses) balanced by buoyancy including the displaced volumes; the incremental lifting of chain uses the *apparent* mass per length (air mass reduced by displaced water).

## 4 Numerical design (step-by-step)

We adopt the EngineeringToolBox numbers (see bibliography):

- Paraffin (volumetric cubic expansion):  $\beta_p = 590 \times 10^{-6} \text{ K}^{-1}$ .
- Water density at 20 °C:  $\rho_w(20^\circ\text{C}) = 998.21 \text{ kg/m}^3$  and its volumetric thermal expansion at 20 °C is  $\beta_w = 2.07 \times 10^{-4} \text{ K}^{-1}$ .
- Paraffin density:  $\rho_p \approx 900 \text{ kg/m}^3$ .
- Steel density:  $\rho_{\text{steel}} \approx 7850 \text{ kg/m}^3$ .
- Lead density:  $\rho_{\text{lead}} \approx 11340 \text{ kg/m}^3$ .
- Chain linear mass (user):  $m_{\text{air,chain}} = 0.083 \text{ g/cm} = 0.0083 \text{ kg/m}$ .
- Desired sensitivity:  $dz/dT = 3 \text{ mm/}^\circ\text{C} = 0.003 \text{ m/K}$ .

### Effective chain linear mass in water

$$m'_{\text{chain}} = m_{\text{air,chain}} \left( 1 - \frac{\rho_w}{\rho_{\text{steel}}} \right) = 0.0083 \frac{\text{kg}}{\text{m}} \left( 1 - \frac{998.21}{7850} \right).$$

Numerically,

$$m'_{\text{chain}} \approx 0.0072446 \text{ kg/m}.$$

### Volume required to achieve 3 mm/°C

Use the boxed formula:

$$V = \frac{m'_{\text{chain}} (dz/dT)}{\rho_w (\beta_p - \beta_w)}.$$

Substitute the numbers (SI units):

$$V \approx \frac{0.0072446 \cdot 0.003}{998.21 \cdot (5.90 \times 10^{-4} - 2.07 \times 10^{-4})}$$
$$V \approx 5.6848 \times 10^{-5} \text{ m}^3 = 56.85 \text{ cm}^3.$$

### If the block is a square prism of thickness 3 cm

If the prism thickness (depth in the page) is  $t = 3 \text{ cm} = 0.03 \text{ m}$  and the square face side is  $a$ , then

$$V = a^2 t \quad \Rightarrow \quad a = \sqrt{\frac{V}{t}}.$$

Numerically,

$$a = \sqrt{\frac{56.8477 \text{ cm}^3}{3 \text{ cm}}} \approx 4.353 \text{ cm}.$$

So the square prism cross-section should be about  $4.35 \text{ cm} \times 4.35 \text{ cm}$  with thickness 3 cm.

### Geometry for a 20 cm vessel

We require the system to fit in a vessel with total internal height  $H_{\text{vessel}} \leq 20 \text{ cm}$ . Choose the float centered at room temperature (so block center at mid-height 10 cm). For the rotated square the vertical distance from center to a vertex (half the diagonal) is

$$\Delta_v = \frac{a}{\sqrt{2}} \approx \frac{4.353 \text{ cm}}{1.4142} \approx 3.078 \text{ cm}.$$

Therefore the bottom vertex is at vertical position  $10 - 3.078 = 6.922 \text{ cm}$  above the vessel bottom; the top vertex is at  $10 + 3.078 = 13.078 \text{ cm}$ . Both lie well within the 0–20 cm vessel height.

### Chain length and lead ballast (approximate)

For compactness and some slack, choose a chain free length between the wall anchor (near the side) and the block bottom vertex of

$$L_{\text{chain}} \approx 20 \text{ cm} = 0.20 \text{ m}.$$

This choice provides ample slack to form a U without contacting the vessel bottom; it fits readily within a small dish of radius a few cm. (If your vessel width is significantly larger or smaller, choose a different  $L$ ; the method below carries through.)

The static (nominal) equilibrium (block centered at room temperature) requires the buoyant force to support the block, ballast and the portion of chain weight transmitted to the block. Approximating that the float supports roughly half of the chain segment's weight (anchor and block are roughly at the same nominal height, symmetric sag assumption), we compute the required lead mass (mass in air) so that:

$$\rho_w V = m_{\text{paraffin}} + m_{\text{lead,air}} \left(1 - \frac{\rho_w}{\rho_{\text{lead}}}\right) + \frac{1}{2} m_{\text{chain,air,segment}},$$

which rearranges to

$$m_{\text{lead,air}} = \frac{\rho_w V - m_{\text{paraffin}} - \frac{1}{2} m_{\text{chain,air,segment}}}{1 - \rho_w / \rho_{\text{lead}}}.$$

Using  $L_{\text{chain}} = 0.20$  m so  $m_{\text{chain,air,segment}} = 0.0083 \cdot 0.20 = 0.00166$  kg,

$$m_{\text{paraffin}} = \rho_p V \approx 900 \cdot 5.6848 \times 10^{-5} \approx 0.05116 \text{ kg}.$$

Then the numerical result is

$$m_{\text{lead,air}} \approx 0.00521 \text{ kg} \approx 5.21 \text{ g}.$$

So pick a small lead pellet of mass  $\approx 5.2$  g (volume  $\approx 0.46 \text{ cm}^3$  at lead density). When submerged the lead's buoyancy is accounted for in the formula above (the factor  $1 - \rho_w / \rho_{\text{lead}}$  corrects for lead buoyancy).

## 5 Summary of recommended dimensions and masses

- Paraffin volume  $V$  (to obtain  $3 \text{ mm}/^\circ\text{C}$ ):  $\boxed{56.85 \text{ cm}^3}$ .
- Square prism (thickness  $t = 3 \text{ cm}$ ): side  $a \approx \boxed{4.35 \text{ cm}}$ .
- Block center at mid vessel (10 cm from bottom). Bottom vertex at  $\approx 6.92 \text{ cm}$  above bottom; top vertex at  $\approx 13.08 \text{ cm}$ .
- Chain length between anchor and block (practical choice):  $L_{\text{chain}} \approx \boxed{20 \text{ cm}}$  (adjust if vessel horizontal dimensions differ).
- Chain linear mass (user):  $0.083 \text{ g/cm} \equiv 8.3 \text{ g/m}$ . Effective (apparent) mass in water:  $\approx 7.24 \text{ g/m}$ .
- Lead ballast (mass in air):  $\boxed{5.2 \text{ g}}$  (ballast volume  $\approx 0.46 \text{ cm}^3$ ).

## 6 Uncertainty and sensitivity to paraffin's expansion coefficient

Paraffin wax is a complex commercial mixture; its thermal expansion depends strongly on exact composition and crystal structure. EngineeringToolBox linear expansion tables show a *wide* range for paraffin (linear coefficient listed in a wide band), and measured paraffin volumetric changes over temperature are known to vary substantially between grades and through phase transitions near the melting region. For that reason the single number  $\beta_p = 5.90 \times 10^{-4} \text{ K}^{-1}$  used above is an engineering estimate, not a precision constant.

### Effect on required volume

The required volume scales approximately as

$$V \propto \frac{1}{\beta_p - \beta_w}.$$

Thus errors in  $\beta_p$  map nonlinearly into  $V$ . Using a plausible paraffin volumetric range (corresponding to linear-range data  $106\text{--}480 \times 10^{-6}$ , i.e. volumetric  $\sim 318 \times 10^{-6}$  to  $1440 \times 10^{-6}$ ), the required block volume for the same sensitivity would span roughly:

$$V(\beta_p = 318 \times 10^{-6}) \approx 196 \text{ cm}^3, \quad V(\beta_p = 1440 \times 10^{-6}) \approx 17.7 \text{ cm}^3.$$

That is a factor of  $\sim 11$  between the extreme values; equivalently, a moderate fractional uncertainty in  $\beta_p$  (tens of percent) produces an order-of-magnitude uncertainty in  $V$ . In practice this means:

- If you require good absolute calibration, measure  $\beta_p$  for the exact paraffin batch (simple dilatometry or submersed-volume measurement with precise thermometer) and then recalculate  $V$  from the boxed formula.
- If you only need approximate sensitivity, the design above (about  $57 \text{ cm}^3$ ) is a reasonable engineering starting point for typical commercial paraffins; but expect the actual mm/deg response to vary unless the paraffin grade is characterized.

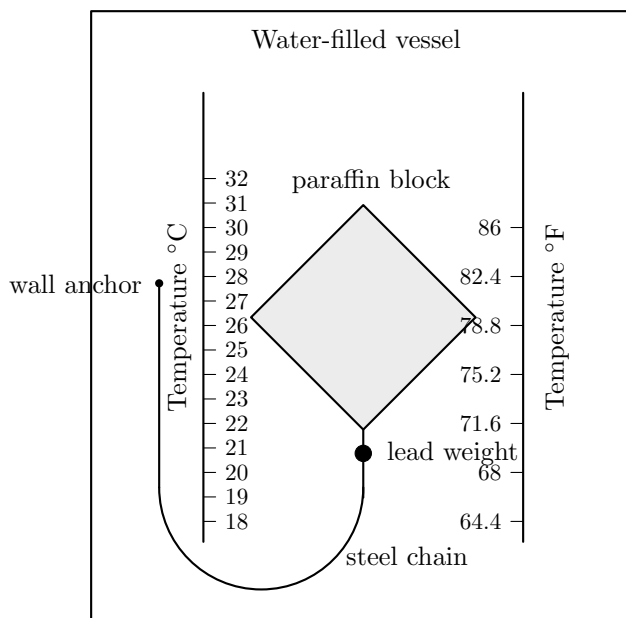
## 7 Practical notes and small-design recommendations

- Use a short chain (20 cm chosen here) and test the static equilibrium in water: adjust lead mass slightly to center the block at room temperature.
- The chain attachment at the bottom vertex should be small and smooth so it does not puncture paraffin; consider a small stainless-steel eyelet embedded in the paraffin before it solidifies.
- For precision reading, mount a fine scale on the outside of the vessel and calibrate the block position (center) with a standard thermometer; if the paraffin is replaced, re-calibrate.
- If you desire a smaller vertical travel per degree (less than  $3 \text{ mm}/^\circ\text{C}$ ), reduce the block volume or use a chain with heavier linear mass; conversely, use a larger block or lighter chain for more motion per degree.

## 8 Materials table (values used in the design)

Property	Symbol	Value (used)	Source
Paraffin volumetric expansion	$\beta_p$	$5.90 \times 10^{-4} \text{ K}^{-1}$	EngineeringToolBox (solids table)
Water density (20 °C)	$\rho_w$	$998.21 \text{ kg/m}^3$	EngineeringToolBox (water table)
Water volumetric expansion (20 °C)	$\beta_w$	$2.07 \times 10^{-4} \text{ K}^{-1}$	EngineeringToolBox (water table)
Paraffin density	$\rho_p$	$900 \text{ kg/m}^3$ (typical)	EngineeringToolBox (specific gravity)
Steel density	$\rho_{\text{steel}}$	$7850 \text{ kg/m}^3$	EngineeringToolBox (metals)
Lead density	$\rho_{\text{lead}}$	$11340 \text{ kg/m}^3$	EngineeringToolBox (metals)
Chain mass per length (air, user)	$m_{\text{air,chain}}$	$0.083 \text{ g/cm}$	user input

## 9 Schematic Diagram



## 10 Concluding remarks

The analysis above gives a practical, buildable starting point: a paraffin block of roughly 57 cm<sup>3</sup> (4.35 cm square face, 3 cm thickness), with a ~20 cm steel chain and about 5.2 g lead ballast will center at room temperature and give roughly 3 mm/°C response (using the EngineeringToolBox material numbers). The most important real-world uncertainty is the paraffin volumetric coefficient  $\beta_p$ . I strongly recommend measuring the actual paraffin sample's expansion (simple dilatometer or submerged-volume test) before final machining if you need accurate calibration; otherwise the device will work qualitatively as a small, continuous thermometer but absolute scale may shift.

## References

- [1] P. Loyson, "Galilean Thermometer Not So Galilean," *Journal of Chemical Education*, **89** (9), 1139–1140 (2012). doi: <https://pubs.acs.org/doi/10.1021/ed200793g>.
- [2] S. Ogden, L. Klintberg, G. Thornell, K. Hjort, and R. Boden, "Review on miniaturised paraffin phase change actuators, valves, and pumps," *Microfluidics and Nanofluidics*, **17**(1), 53–71 (2014). doi: <https://doi.org/10.1007/s10404-013-1289-3>.
- [3] EngineeringToolBox, "Solids - Volume Temperature Expansion Coefficients". Available: [https://www.engineeringtoolbox.com/volum-expansion-coefficients-solids-d\\_1894.html](https://www.engineeringtoolbox.com/volum-expansion-coefficients-solids-d_1894.html).
- [4] EngineeringToolBox, "Water Density, Specific Weight and Thermal Expansion Coefficients - Temperature and Pressure Dependence". Available: [https://www.engineeringtoolbox.com/water-density-specific-weight-d\\_595.html](https://www.engineeringtoolbox.com/water-density-specific-weight-d_595.html).
- [5] EngineeringToolBox, "Solids and Metals - Specific Gravities". Available: [https://www.engineeringtoolbox.com/specific-gravity-solids-metals-d\\_293.html](https://www.engineeringtoolbox.com/specific-gravity-solids-metals-d_293.html).

- [6] EngineeringToolBox, “Metals and Alloys - Densities”. Available: [https://www.engineeringtoolbox.com/metal-alloys-densities-d\\_50.html](https://www.engineeringtoolbox.com/metal-alloys-densities-d_50.html).
- [7] EngineeringToolBox, “Linear Thermal Expansion Coefficients of Materials”. Available: [https://www.engineeringtoolbox.com/linear-expansion-coefficients-d\\_95.html](https://www.engineeringtoolbox.com/linear-expansion-coefficients-d_95.html).
- [8] M. Kraiem, M. Karkri, M. Fois, P. Sobolciak, “Thermophysical Characterization of Paraffins versus Temperature for Thermal Energy Storage”, *Buildings*, 2023, 13, 877. <https://www.mdpi.com/2075-5309/13/4/877>.
- [9] V. Silva-Nava, “Thermophysical Characterization of Paraffin Wax Based on Mass-Accommodation Methods”, *Molecules* 2022, 27(4), 1189. <https://www.mdpi.com/1420-3049/27/4/1189>.



## AI Prompts

Act as an agent and help me with this transport problem step by step.

Return the answer as a 5 to 8 page report as a latex script in Chat.

Do not use unicode characters. LaTeX should compile without error in TeXShop.

In this problem we are designing a variant on a Galileo thermometer. A block of paraffin immersed in water is attached to a flexible steel chain. The other end of the chain is attached to the wall of a vessel at approximately the same vertical level as the center of the block. Between these two points the limp chain hangs downward in the shape of a U. As the temperature changes, the float rises or falls and supports a greater or lesser amount of the weight of the chain (with the remainder being supported by the anchor point on the wall). Because the paraffin block is very light, it is useful to attach a small weight (lead) to the bottom of the block so that the immersed block is centered in the vessel at room temperature. You are to determine the parameters and mode of operation of this thermometer.

The report should contain the following elements:

A history of the Galileo thermometer with references, comparing to the proposed design.

An analysis of the operation of the proposed thermometer including equations.

The behavior of the device depends primarily on the different coefficient of thermal expansion of solid paraffin and water at room temperature. Use values from [engineeringtoolbox.com](http://engineeringtoolbox.com) for your calculations.

Considering the coefficient of thermal expansion of all materials at room temperature, if the steel chain has a mass of 0.083 g/cm in air (note: correct for buoyancy of water) what should be the volume of the paraffin block so that it has a vertical motion of 3mm/degree C?

If the block is a square of thickness 3cm, what should be the other dimensions?

The system should be contained in a vessel that is no more than 20 cm in height. Based on this, approximately what should be the length of the chain and what should be the mass of the lead weight (correct for buoyancy of water) so that everything fits?

Comment on the uncertainty of the coefficient of thermal expansion of paraffin, and how errors would affect the temperature sensitivity of the thermometer. Provide an estimate of the uncertainty in this value with references.

Include a TikZ script of a figure of the design. The paraffin block should be a square with one vertex down with the chain and weight attached to this vertex. The temperature scale should align with the vertices on the side, the one on the left in degrees C and the one on the right in degrees F.

Provide a table of the material properties used in the design with validated sources.

Provide a bibliography in the "thebibliography" format with verified url or doi.

The latex script should compile without error in TeXShop.

## Corrections

The report produced by the above prompt was quite good, with values correctly calculated. In section 7 (practical notes) the effect of volume and chain weight on travel were backwards in chat's version, this was changed manually.

The TikZ figure produced by this prompt was a disaster. An earlier figure generated from a verbal prompt was used instead. The U shaped chain from that figure was modified and the coordinates of a number of the labels were adjusted to improve clarity.

The engineeringtoolbox.com references and url's were correct. The last two references existed, however the url (doi) for each were manually repaired.

The short historical note was updated to include an interesting Journal of Chemical Education article, and additional citations (hallucinations) were added by Chat. These were fixed or removed.

## Prompting Strategy

The approach used was to start by simply feeding in the problem statement to see what Chat would produce. It was noticed by looking at the "thinking" results that it was having a lot of trouble finding the coefficient of thermal expansion of the various materials, particularly paraffin. This was degrading its response on the rest of the report as well (there are a limited number of web calls allowed). It was decided to direct it to the engineeringtoolbox.com site to get consistent values, but also to do a sensitivity analysis of the effect of errors in the paraffin value. An additional error noted was that it was neglecting (about half the time) to correct the chain weight for the buoyancy of water, so this was specifically added into the prompt. The 20cm in height for the vessel was also added so that the final design would be more reasonable. The final prompt (given above) was inserted after a "start fresh" command, and then modifications of the report were made as described above.